Over Networks $H_{\infty}$ Filtering for Discrete Singular Jump Systems with Interval Time-varying Delay

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Abstract: This paper deals with the robust $H_{\infty}$ filtering for discrete singular systems with jump parameters and interval time-varying delay, whose system mode is transmitted through an unreliable network. The class of systems under consideration is more general and covers the singular jump systems with mode-dependent and mode-independent as two special cases, over networks a novel delay-dependent and partially mode-dependent filter is established via using a mode-dependent Lyapunov function and a finite sum inequality based on quadratic terms. The corresponding filter parameters can be obtained by solving a set of linear matrix inequalities without decomposing the original system matrix. The proposed linear robust filter ensuring that the filtering error singular jump system is to be regular, causal, stochastically stable and satisfies $H_{\infty}$ performance. In addition, two numerical examples are given to illustrate the effectiveness of the proposed approach.

Keywords: Delay-dependent, discrete singular jump systems, $H_{\infty}$ filtering, interval time-varying delay, linear matrix inequality (LMI), partially mode-dependent.

1. INTRODUCTION

As a special class of hybrid systems, Markovian jump systems (MJSs) have been attracting extensive research attention over the past decades due to their widely practical applications in manufacturing systems, power systems, aerospace systems and networked control systems. The control and filtering problems related to MJSs with or without time-delay have been fully investigated, see e.g., [1-8] and the references therein. Recently, many notions and results in state-space jump systems, such as stability and stabilization [9,10], $H_{\infty}$ control [11-13], filtering problem researching [14-16] and so on. When Markovian jump parameters appear, it should be pointed out that the problem for SMJSs is much more complicated than that of state-space jump systems, because it requires to consider not only stability and modes switching, but also regularity and impulse elimination (for continuous-time singular systems) or causality (for discrete-time singular systems) simultaneously, while the latter two do not appear in regular ones.

Filtering is a class of important approaches to estimate the state information when the system plant is disturbed. Currently there are many approaches proposed for filter design, such as Kalman filtering [17,18], $H_{\infty}$ filtering [19-21], $l_2–l_{\infty}$ filtering [22-24] etc. In the $H_{\infty}$ filtering, the input is supposed to be an energy signal and the corresponding energy-to-energy gain can be minimized. Recently, many results on the $H_{\infty}$ mode-dependent and mode-independent filtering for MJSs have been presented in [25-35] and the references therein. However, these results on the filtering for the MJSs require critical assumption on the accessibility of the jumping mode and main classified to three types of stochastic filters, which the first type is mode-dependent filtering with completely known transition modes of systems [25,26]; the second type is mode-independent filtering design ignoring mode information in the filter construction [27,28]; the last one is with partially unknown transition probabilities of jump mode, such as researched in [30-32].

In practical applications, the aforementioned assumptions may sometimes be impossible to satisfy, such as the networked control systems (NCSs) [33,34,40-42], the introduction of communication networks in feedback control loops complicates the system analysis and synthesis, which also introduces new interesting and challenging problems. For the cases that the underlying NCSs is an MJS, both system state and mode are transmitted, when the system mode transmitted through networks suffers being lost and observed simultaneously, it is said that mode-dependent method is too ideal, whereas mode-independent algorithm is too absolute. Thus, both of the two extreme filter design methods are not suitable to the case where the system mode is available to a filter with some probabilities and time-varying delay through unreliable networks. Although in [32], a new filtering method was established for a class of MJSs, it should be pointed out that discrete-time SMJSs with time-varying delay are much more important...
than their continuous-time counterparts in our digital world. The problem of delay-dependent robust $H_\infty$ filtering for discrete-time SMJSs with system mode available to a filter through unreliable networks has not been fully investigated. It is, therefore, the main purpose of the present research to shorten such a gap by making the first attempt to deal with the delay-dependent and partially mode-dependent $H_\infty$ filtering problem of a class of discrete-time SMJSs over networks.

This paper is concerned with the delay-dependent and partially mode-dependent robust $H_\infty$ filtering problem for a class of discrete-time SMJSs over networks. The considered systems are more general and cover the singular Markovian systems with completely known or complete-ly unknown transition modes as two special cases. First in contrast with the traditionally mode-dependent or mode-independent filtering design with interval time-varying delay. Based on this, a new delay-dependent and partially mode dependent filter is established via using a mode-dependent Lyapunov function and a finite sum inequality based on quadratic terms. The suitable filter parameters which is solved by employing the LMIs technique and without decomposing the original system matrix. The desired filter which guarantees the admissibility and the $H_\infty$ performance of the corresponding filtering error system. Last, two numerical examples are provided to illustrate the effectiveness of the developed theoretical results.

**Notations:** Throughout this paper, for real symmetric matrices $X$ and $Y$, the notation $X\geq Y$ (respectively, $X>Y$) means that the matrix $X-Y$ is semi-positive definite(respectively, positive definite); $R^n$ and $R^{m\times n}$ denote the n-dimensional Euclidean space and the set of all $m \times n$ real matrices, respectively; $I$ is the identity matrix with appropriate dimension; the superscript $T$ represents the transpose of a matrix; $\|X\|$ refers to Euclidean norm of the vector $X$, $Z$ denotes the set of non-negative integer numbers; $\epsilon[\bullet]$ denotes the mathematical expectation, $\text{diag}(\bullet)$ means block diagonal matrix, $He\{M\}$ stands for $M+M^T$, and $*$ denotes the symmetric term in a symmetric matrix.

**2. PROBLEM STATEMENTS AND PRELIMINARIES**

In this paper, fix a probability space $(\Omega, F, P)$ and consider the following discrete random jump systems with interval time-varying delay

$$
\begin{align*}
E\{x(k+1)\} &= A(\theta(k))x(k) + A_d(\theta(k))x(k-\tau(k)) + B(\theta(k))\omega(k), \\
y(k) &= C(\theta(k))x(k) + C_d(\theta(k))x(k-\tau(k)) + D(\theta(k))\omega(k), \\
z(k) &= L(\theta(k))x(k) + L_d(\theta(k))x(k-\tau(k)) + G(\theta(k))\omega(k), \\
x(k) &= \phi(k), \quad k = -\bar{T}, -\bar{T}+1, \ldots, 0,
\end{align*}
$$

(1)

where $x(k) \in R^n$ is the system state, the matrix $E \in R^{mn}$ may be singular, we shall assume that $\text{rank}(E) = r \leq n$. $y(k) \in R^p$ is the measurement vector, $\omega(k) \in R^m$ is the disturbance input which belongs to $L_2[0, \infty)$. $z(k) \in R^p$ is the signal to be estimated, $\phi(k)$ is a known initial condition, where $(\theta(k), k) \in Z$ are discrete Markov chains that takes values in $l = \{1, 2, \ldots, N\}$. Its transition probability matrix is $\Lambda = \{\lambda_{ij}\}$, which is defined

$$
\lambda_{ij} = P(\theta(k+1) = j | \theta(k) = i) \geq 0, \quad \forall i, j \in l.
$$

and 

$$
\sum_i \lambda_{ij} = 1 \quad \text{for all } i \in l.
$$

For each possible value of $\theta(k) = h \in l$, $A_h, A_d, B_h, C_h, C_d, D_h, L_h, L_d$ and $G_h$ are known constant matrices with appropriate dimensions. $(\tau(k)$ is time-varying delay and satisfies

$$
0 < \tau(k) \leq \bar{T} < \infty,
$$

(2)

where $\tau$ and $\bar{T}$ are known positive integers.

The objective of this paper is to develop a novel partially mode-dependent filter over networks with state-space realization of the following form:

$$
\begin{align*}
x_f(k+1) &= A_f x_f(k) + B_f y(k) + \alpha(k)(A_f(\theta(k))x_f(k) + B_f(\theta(k))y(k)), \\
z_f(k) &= C_f x_f(k) + D_f y(k) + \alpha(k)(C_f(\theta(k))x_f(k) + D_f(\theta(k))y(k))
\end{align*}
$$

(3)

where $x_f(k) \in R^n$ filter state, and $z_f \in R^p$ is the estimation single, $A_f \in R^{mn}$, $B_f \in R^{m \times p}$, $C_f \in R^{m \times n}$, $D_f \in R^{m \times p}$, $A_f(\theta(k)) \in R^{mn}$, $B_f(\theta(k)) \in R^{m \times p}$, $C_f(\theta(k)) \in R^{m \times n}$ and $D_f(\theta(k)) \in R^{m \times p}$ are filter parameters to be determined.

Notice that filter (3) and system (1) have the same mode, $\theta(k)$ is assumed to be known in this paper. $\alpha(k)$ is an indicator function described as: if $\theta(k)$ transmitted successfully, then $\alpha(k) = 1$, otherwise $\alpha(k) = 0$. Assume $\alpha(k)$ is a Bernoulli distributed sequence with

$$
Pr[\alpha(k) = 1] = \epsilon[\alpha(k)] = \alpha, \quad Pr[\alpha(k) = 0] = 1 - \alpha,
$$

(4)

where $\alpha$ is a constant $0 \leq \alpha \leq 1$. Furthermore, we have

$$
\epsilon[\alpha(k) - \alpha^2] = 0, \quad \beta^2 := \epsilon[(\alpha(k) - \alpha)^2] = \alpha(1-\alpha).
$$

(5)

In addition, the two random processes $\alpha(k)$ and $\theta(k)$ are assumed to be independent. Define the augmented state vector $\bar{x}(k) = [x(k)^T \quad x_f(k)^T]^T$, and the estimation error $\bar{z}(k) = z(k) - z_f(k)$, for simplicity, let the mode at time $k$ be $i$, that is $\theta(k) = i$. Then the filtering error singular jump system is obtained as follows:

$$
\begin{align*}
E\{\bar{x}(k+1)\} &= A_{\theta_i} \bar{x}(k) + A_{\theta_d} K_{x_i} (\bar{z}(k-\tau(k)) + \bar{B}_i \omega(k)) \\
&+ \alpha(k)(A_{\theta_i} x_f(k) + A_{\theta_d} K_{x_f} (\bar{z}(k-\tau(k)) + \bar{B}_f \omega(k)), \\
\bar{z}(k) &= L_{\theta_i} \bar{x}(k) + L_{\theta_d} K_{x_i} (\bar{z}(k-\tau(k)) + \bar{D}_i \omega(k) \\
&+ \alpha(k)(L_{\theta_i} x_f(k) + L_{\theta_d} K_{x_f} (\bar{z}(k-\tau(k)) + \bar{D}_f \omega(k)), \\
\bar{x}(k) &= [\phi(k)^T \quad 0]^T, \quad k = -\bar{T}, -\bar{T}+1, \ldots, 0,
\end{align*}
$$

(6a)

(6b)