APPLICATIONS OF THE EXTENDED TEST APPROACH TO
(2 + 1)-DIMENSIONAL GARDNER EQUATION

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Based on the extended test approach (ETA), we investigated the nonlinear evolution equations, namely, (2 + 1)-dimensional Gardner equation. We aimed to obtain some exact breather-type and periodic-type soliton solutions for this model. These results show that the extended test technique together with the bilinear method is a simple and effective method to seek exact solutions for nonlinear evolution equations. The properties of some periodic-like and soliton-like solution for this system are shown by some figures.

Key words: Extended test approach (ETA); (2 + 1)-dimensional Gardner equation; breather-type soliton solutions; periodic-type soliton solutions.

1. INTRODUCTION

Many phenomena in physics and other fields are often described by nonlinear partial differential equations (PDEs). When we want to understand the physical mechanism of phenomena in nature, described by nonlinear PDEs, exact solutions have to be explored. In the past few decades, many powerful and systematic methods have been developed [1-3]. The extended test approach (ETA), which is based on the bilinear form of the NLEEs, is a fairly effective method to seek periodic solitary wave solutions of integrable equations [4, 5]. By using this technique, the new solitary wave, periodic solitary wave, periodic kink-wave and breather-type soliton solutions of some integrable equations, such as KdV equation, modified KdV equation, Zakharov
equation, (2 + 1)-dimensional Ginzburg-Landau equation, (3 + 1)-dimensional KP equation, (2 + 1)-dimensional Boussinesq equation and so on, are obtained [6-15].

The purpose of this paper is to present solutions of the (2+1)-dimensional Gardner equation by using ETA. Some conclusions are given.

2. DESCRIPTION OF THE EXTENDED TEST APPROACH (ETA) METHOD

We consider a general form of a higher dimensional nonlinear evolution equation

\[ P(u, u_x, u_y, u_z, u_{xx}, u_{yy}, u_{zz}, \ldots) = 0, \]  

where \( u = u(x, y, z, t) \) and \( F \) is a polynomial in \( u \) and its derivatives.

The basic idea of the extended two-wave method can be expressed in the following form:

**Step 1:** By Painlevé analysis, a transformation is carried out

\[ u = T(f), \]  

where \( f \) is a new unknown function.

**Step 2:** By using (2), eq. (1) can be converted into Hirota bilinear form

\[ G(D_t, D_x, D_y; f, f) = 0, \]  

where the D-operator [16] is defined by

\[ D_x^n D_t^m f(x, t) \cdot g(x, t) = \left( \frac{\partial}{\partial x} - \frac{\partial}{\partial x'} \right)^m \left( \frac{\partial}{\partial t} - \frac{\partial}{\partial t'} \right)^n \]  

\[ \left[ f(x, t) g(x', t') \right] \bigg|_{x'=x, t'=t} \]  

**Step 3:** To seek the soliton solution of eq. (3), we assume the standard ansatz in ETA [4, 5, 16]

\[ f = \nu_1 e^{-\zeta_1} + \nu_2 \cos(\zeta_2) + \nu_1 e^{\zeta_1}, \]  

for (2 + 1)-dimensional system, where \( \zeta_i = a_i x + b_i y + c_i t + \zeta_i^0 \) and \( a_i, b_i, c_i \) and \( \nu_i, i = 1, 2 \) and \( \nu_{-1} \) are unknown constants to be determined later.