A Note on Determining Parameter Redundancy in Age-Dependent Tag Return Models for Estimating Fishing Mortality, Natural Mortality and Selectivity

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Jiang et al. (JABES 12:177–194, 2007) present models for tag return data on fish. They examine whether the models are parameter redundant, but need to resort to numerical methods as symbolic methods were sometimes found to be intractable. Also, their results are only applicable for a specified number of years of tagging data and age-classes. Here we show how symbolic methods can in fact be used and also how conclusions apply to any number of years of tagging data and age-classes.

This article has supplementary material online.

Key Words: Derivative matrix; Ecology; Exhaustive summaries; Identifiability; Maple.

1. INTRODUCTION

Jiang et al. (2007) present new models for tag return data on fish. The important advance in the paper is the incorporation of age-dependence in the models. They are based on the probability that a fish tagged at age \( k \), released in year \( i \), and harvested and returned in year \( j \) has the form

\[
P_{ijk} = \begin{cases} 
(1 - e^{-F_j Sel_{k+j-i-M}}) \frac{F_j Sel_k \lambda}{F_j Sel_k + M}, & j = i, \\
(\prod_{v=i}^{j-1} e^{-F_v Sel_{k+v-i-M}}(1 - e^{-F_j Sel_{k+j-i-M}}) \frac{F_j Sel_{k+j-i-M}}{F_j Sel_{k+j-i-M}}, & j > i,
\end{cases}
\]

where the parameters are: \( F_j \) the instantaneous fishing mortality rate for fully recruited fish, \( Sel_a \) the selectivity coefficient for fish aged \( a \) (with fish being fully recruited at age \( a_c \), so that \( Sel_{a_c} = 1 \) for \( a > a_c \)), \( M \) the instantaneous natural mortality rate and \( \lambda \) the reporting
probability for dead fish. The parameters $M$ and $\lambda$ can depend on year and age, in which case $M_{y,a} = M_1^Y M_{a}^A$ with $M_1^Y = 1$, with a similar parameterization for $\lambda$. Henceforth we shall refer to their paper by “JPBH3”.

JPBH3 examine whether their models are parameter redundant, using the symbolic algebra method of Catchpole and Morgan (1997). The number of estimable parameters is equal to the symbolic rank of the derivative matrix $D = \left[ \frac{\partial P_{jk}}{\partial \theta} \right]$, where $P_{ij}$ refers to each of the non-zero $P_{ij}$ taken in turn (Catchpole and Morgan 1997). It is not possible to evaluate the rank of $D$ for several of JPBH3’s models; they are structurally too complex and the symbolic algebra package Maple runs out of memory. JPBH3 use numerical methods instead.

Recently, Cole and Morgan (2009) presented a more general approach to determining parameter redundancy, in which the derivative matrix arises from differentiating what is called an exhaustive summary, which can be simplified using reparameterization. In this paper we use their approach to show how symbolic algebra can be used to determine whether the models of JPBH3 are parameter redundant.

2. DETERMINING THE PARAMETER REDUNDANCY STATUS OF THE MODEL

We start by considering the case when there are 2 age classes with $a_c = 2$, 4 years of tagging and 4 years of recovery. The full model, which incorporates year and age dependence, in this instance has 16 parameters:

$$\theta = \left[ F_1, F_2, F_4, \text{Sel}_1, \text{Sel}_2, M_2^Y, M_3^Y, M_4^Y, M_2^A, M_3^A, M_4^A, \lambda_2, \lambda_3, \lambda_4, \lambda_1, \lambda_2 \right].$$

Maple cannot calculate the rank of the derivative matrix $D = \left[ \frac{\partial P_{ij}}{\partial \theta} \right]$. Instead, we use reparameterization to find the new exhaustive summary:

$$r = \begin{bmatrix}
F_1 \text{Sel}_1 + M_1^A \\
F_1 \text{Sel}_2 + M_2^A \\
F_2 \text{Sel}_1 + M_2^Y M_1^A \\
F_2 \text{Sel}_2 + M_2^Y M_2^A \\
F_2 + M_2^Y M_2^A \\
F_3 \text{Sel}_1 + M_3^Y M_1^A \\
F_3 \text{Sel}_2 + M_3^Y M_2^A \\
F_3 + M_3^Y M_2^A \\
F_4 \text{Sel}_1 \lambda_1^A \\
F_4 \text{Sel}_2 \lambda_2^A \\
F_4 \lambda_2^A \\
F_4 \text{Sel}_1 \lambda_3^A \lambda_1^A \\
F_4 \text{Sel}_1 \lambda_4^A \lambda_1^A (1 - \exp(-F_4 \text{Sel}_1 - M_4^Y M_1^A)) / (F_4 \text{Sel}_1 + M_4^Y M_1^A) \\
F_4 \text{Sel}_1 \lambda_4^A \lambda_1^A (1 - \exp(-F_4 \text{Sel}_2 - M_4^Y M_2^A)) / (F_4 \text{Sel}_2 + M_4^Y M_2^A) \\
F_4 \lambda_4^A \lambda_2^A (1 - \exp(-F_4 - M_4^Y M_2^A)) / (F_4 + M_4^Y M_2^A)
\end{bmatrix}.$$ (2.1)