Nonlinear Dynamics of Three-Dimensional Prediction Model for a Flexible Riser Under Linearly Sheared Currents

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Abstract
This paper presents a three-dimensional vortex-induced vibration prediction model for a long flexible riser under linearly sheared currents. Two distributed and coupled van der Pol wake oscillators are utilized to characterize the fluctuating lift and drag coefficients, respectively. It should be noted that geometric and hydrodynamic nonlinearities are also considered in our model. Numerical simulations by finite element method are carried out to solve the highly coupled nonlinear fluid–structure interaction equations. Firstly, modal analysis is performed to obtain the foremost ten natural frequencies of the flexible riser under top-end tension by theoretical and numerical methods, and the results agree very well. Then, nonlinear dynamic analyses are carried out to investigate the effects of linear shear flow on displacements, stresses, modal variations and phase portraits. The results obtained in uniform and linear shear currents are compared in detail. The results indicate that the asymmetric phenomenon along the riser span is more obvious with increasing linear shear velocity and the lock-in phenomenon of IL (in-line) response frequencies with multi-frequency is also observed along riser span. Moreover, it is also revealed that the dynamic responses simultaneously exhibit the standing and travelling wave patterns under linearly sheared currents, and the dynamic responses become more irregular than uniform flow.

Keywords Flexible riser · Linearly sheared current · Wake oscillator · Vortex-induced vibration · Fluid–structure interaction

List of Symbols

\( L \) Length of pipe (m)
\( D \) Outer diameter (m)
\( d \) Inner diameter (m)
\( \rho_o \) Outer fluid density (kg/m\(^3\))
\( m \) Pipe mass per unit length (kg/m)
\( m_a \) Additional fluid mass per unit length (kg/m)
\( U \) Cross-flow velocity (m/s)
\( E \) Elasticity modulus (Pa)
\( c \) Damping coefficient (N/s)
\( I \) Moment of inertia (m\(^4\))
\( A_t \) Section area of pipe (m\(^2\))
\( T_t \) Top pre-tension (N)
\( T \) Static effective tension (N)
\( F_x \) \( x \) direction hydrodynamic force (N)
\( F_y \) \( y \) direction hydrodynamic force (N)
\( F_z \) \( z \) direction hydrodynamic force (N)
\( u, v, w \) Displacements components (m)
\( \dot{\square} \) Differentiation about time (t)
\( \overset{'}{\square} \) Differentiation about axial coordinate (z)

1 Introduction

Vortex-induced vibration (VIV) of a long flexible riser pipe placed within ocean currents generates several intriguing fluid structure interaction (FSI) phenomena which are widely found in ocean engineering, including risers, mooring cables and pipelines [1–6]. The VIVs are the major reason of fatigue damage and reduction in lifetime for these structures. The in-line (IL), cross-flow (CF) and axial (AX) dynamic response resulting from VIV have significant effect on displacements and stresses related to fatigue and lifetime of the flexible riser [7–9].

Over the past several decades, the VIVs have been paid considerable attention to VIV modelling, simulations and experiments for elastically supported rigid cylinders and long
flexible cylinders [10–13]. The comprehensive reviews about VIV have been investigated by a few researchers [14–18] in the last few years. As for the VIV prediction models of flexible long cylinders under uniform flow, Dai et al. [19–21] studied VIV dynamics of fluid-conveying risers under different excitations including pulsating fluid and base excitations on CF direction, and the results illustrated that internal flow velocity has an important effect on the dynamic responses. A model with geometric imperfections was examined by Wang et al. [22], and it was concluded that a small buckling displacement occurs at the critical internal flow velocity. A nonlinear CF and AX dynamic coupling model was proposed to analyse the dynamic behaviour of a deepwater riser under a hard suspension characteristic in [23]. To take into account the IL and AX dynamic response, a three-dimensional (3-D) VIV prediction model of a flexible pipe with geometric nonlinearities had been established [24, 25]. Zanganah and Srinil [26] presented a 3-D phenomenological model for the analysis and prediction of VIV of a long flexible pipe under uniform flow and solved the coupled FSI equations by finite difference method (FDM), and results showed that this model can accurately predict the nonlinear dynamic response of flexible risers by comparing experimental data [25]. Based on the model, Yang et al. [26] discussed the effect of Kelvin–Voigt viscoelasticity on the vibration suppression of marine riser. In recent, the 3-D VIV prediction model for a flexible fluid-conveying pipe was proposed in [27].

Nevertheless, the distribution of ocean currents along the vertical riser span is nonuniform in most cases. Chaplin et al. [28] investigated VIVs of a vertical tension riser in a stepped current by experimental method and revealed that IL and CF displacements strongly depend on the modal composition of the motion. Lie and Kaasen [29] performed large-scale model testing of a tensioned steel riser with the length 90 m and the diameter 3 cm in well-defined sheared current, and the results demonstrated that numerous intriguing phenomena were observed, such as irregular riser responses, multi-mode superposition and dual frequency responses. Bourguet et al. [30] studied the IL and CF VIVs of a long flexible cylinder immersed in a linearly sheared oncoming flow by direct numerical simulation (DNS) method, and the results illustrated that standing and travelling wave patterns will simultaneously exist in both the in-line and cross-flow directions and also showed that the wake can lock into different frequencies at various spanwise locations in the case of multi-frequency response and within the lock-in region. The VIVs of a flexible riser of aspect ratio 200 within first a linearly and then an exponentially sheared current are investigated by DNS method [31], and it was revealed that the fluid forces associated with the broadband responses are dominated by high frequencies related to high-wavenumber vibration components. Fully 3-D computational fluid dynamics (CFD) solutions combined with structural models of a tensioned riser were presented to predict riser vortex-induced motion in [32]. Bourguet et al. [33] investigated the mechanisms of broadband VIVs of a flexible riser of aspect ratio 200 immersed in shear flow, with an exponentially varying profile along the span by DNS method, and found that the broadband responses are associated with the distribution of the lock-in condition along the span. Wang and Xiao [34] presented a numerical simulation on vortex-induced vibration of a vertical flexible riser under uniform and linearly sheared currents, and the results are in good agreement with the published experimental data. However, these researches lack of focusing on investigating the theoretical prediction model for flexible risers under nonuniform currents.

Consequently, the prediction model for flexible risers under nonuniform currents has been investigated by a few researchers. Srinil [13] developed a CF VIV prediction model of variable-tension vertical risers under linear shear flow, and the result concluded that numerical predictions provide good qualitative agreement with some published experimental and computational fluid dynamic observations. However, insights into the fully coupled CF, IL and AX VIV prediction model of a flexible riser under linearly sheared currents are still lacking in the literature. Therefore, the present research is aimed at overcoming such model limitation to investigate 3-D dynamic VIV responses of a flexible riser under linear shear flow.

The paper is organized as follows. Considering the geometric nonlinearity, the three-dimensional VIV model of a flexible riser under linearly sheared currents is presented in Sect. 2. Nonlinear hydrodynamic lift and drag forces are discussed in Sect. 3. In Sect. 4, the frequency analyses and nonlinear dynamic responses with different cross-flow velocities under uniform and linear shear flow are investigated in detail. The main findings of the present work are summarized in Sect. 5.

### 2 Three-Dimensional VIV Mathematical Model for the Flexible Riser

In this study, a riser which is perfectly straight at its vertical static equilibrium under the effective weight is fully immersed in water. The flexible riser simply supported at both ends is placed in linearly sheared oncoming currents aligned with \( z \) direction, as depicted in Fig. 1a. It is noteworthy that the riser will move three dimensionally about the section profile in in-line (IL) \( x \), cross-flow (CF) \( y \) and axial (AX) \( z \) direction. In Fig. 1a, the maximum velocities are at the top end of the riser \(( z = L )\) with \( U(L) = U \) and the minimum velocities are at the bottom end of the riser \(( z = 0 )\) with \( U(0) = 0 \).

Following Zanganah and Srinil [24], the nonlinear partial differential equations of 3-D coupled IL, CF and AX motions