HIGH ORDER LAGRANGIAN VELOCITY STATISTICS IN A TURBULENT CHANNEL FLOW WITH $Re_\tau = 80^*$

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Abstract: The scaling properties of high-order Lagrangian velocity structure functions are investigated numerically for a turbulent flow with a friction-velocity based Reynolds number $Re_\tau = 80$. The Lagrangian particles are released from locations of different distances to the wall. The relative scaling exponents $\zeta^L_1(q)$ of the longitudinal velocity component are found to increase with the released distance to the wall and to approach asymptotically to theoretical predictions. However, the scaling exponents $\zeta^L_2(q)$ of the transverse velocity component are smaller than $\zeta^L_1(q)$, indicating a more intermittent nature. Specifically for the release locations in the center region, the relative scaling exponents $\zeta^L_2(q)$ agree very well with theoretical predictions.

Key words: Lagrangian scaling exponents, intermittency, turbulent channel flow

Introduction

The problem of anomalous scaling can be seen as one of the great unsolved problems in the turbulence research[1]. Intermittency, also known as multifractal nature, is a basic feature of the fully developed turbulence, which is usually considered in a Eulerian frame. The so-called intermittency provides corrections to Kolmogorov (1941) (hereafter K41) scaling law of small scale structures of local homogeneous and iso-

tropic turbulent flows. On the other hand, intermittency can be also investigated in a Lagrangian framework, in which the velocity of particles is tracked numerically or experimentally. The knowledge of Lagrangian statistics in fully developed turbulent flows is a key ingredient for Lagrangian stochastic models in diverse contexts, such as turbulent combustion, pollutant dispersion, cloud formation and industrial mixing[2-4].

It is found that the small-scale intermittency of turbulent flows and the dissipation of Lagrangian energy can be studied more efficiently in the Lagrangian framework[5]. More recently, several experimental and numerical studies of Lagrangian turbulence were reported[5-6]. The Lagrangian statistics are found numerically and experimentally to be strong intermittent[5-7]. For example, Xu et al.[7] found that the Lagrangian scaling exponents at high orders are significantly deviated from the K41 predictions, indicating a strong intermittent nature of turbulent flows. Furthermore, the agreement between experiments and
numerical simulations is good for lower-order exponents, but less satisfactory for higher-order exponents. The scaling exponents for the transversal velocity component are found to be more intermittent. It is of great interest to evaluate how the scaling exponents depend on the initial release position of Lagrangian particles.

In this article, we study the high-order moments of the Lagrangian longitudinal and transversal velocity increments $\Delta v = |v(t + \tau) - v(t)|$ in a low Reynolds number turbulent flow with a friction-velocity based Reynolds number $Re_f = 80$. Lagrangian particles are released, respectively, from the buffer layer, the log-law layer and the central region. According to Kolmogorov’s arguments, one has a power law behavior for the Lagrangian velocity structure-function, i.e.,

$$\left\langle \Delta v^\eta (\tau) \right\rangle \sim \tau^{\zeta_\eta(q)}$$

(1)

where $\eta \leq \tau \leq L$, and $\eta$ is the so-called Kolmogorov dissipation scale, $L$ is the integral scale and $\zeta_\eta(q)$ is Lagrangian scaling exponents. For a non-intermittent case, one has $\zeta_\eta(q) = q/2$ for Kolmogorov values. However, due to the intermittent nature of turbulent flows, these exponents deviate from those predicted by Kolmogorov’s theory systematically, indicating intermittency.

Some efforts were made to derive anomalous Lagrangian scaling exponents. Among them, one is based on a multifractal model, which gives the exponents as a Legendre transform of the con-dimension function. When the Lognormal model is adopted for the Eulerian structure function’s exponents, the exponents of Lagrangian structure-function take the form as

$$\zeta_\eta(q) = a - 1 + \sqrt{(1-a)^2 + 4qb - 2bq}$$

(2)

where $a = (2 + \mu)/6$ and $b = \mu/18$, $\mu = 0.23$ is the intermittency parameter. Zybin et al. also proposed a theory of vortex formation based the Navier-Stokes equation to obtain analytically the anomalous scaling. They reported the first substantial advance in the derivation of both Lagrangian and Eulerian scaling exponents based on the Navier-Stokes equation. The exponents of the Lagrangian structure function are (see Zybin et al. for more details)

$$\zeta_\eta(q) = q - \frac{A_{2q}}{2\lambda}$$

(3)

where $A_{2q}$ is the maximal root of the corresponding characteristic equation of vorticity, and $\lambda = A_{q}/2$.

The lognormal model (Eq.(2)) and Zybin et al.’s theoretical prediction (Eq.(3)) will be used to compare our Direct Numerical Simulation (DNS) results and experimental and numerical results reported in recent articles.

1. Numerical method

A direct numerical simulation of a turbulent channel flow at a low Reynolds number is performed with a friction Reynolds number $Re_f = u_\delta / \nu = 80$, where $u_\delta$, $\delta$ and $\nu$ are, respectively, the friction velocity, the half width of the channel and the kinematic viscosity. The domain sizes in the streamwise, wall-normal, and spanwise directions are $180\times160\times630$ in wall units. The corresponding grid numbers in three directions are $96\times65\times64$. The mass conservation equation and the Navier-Stokes (resp. momentum) equations of incompressible fluids are as follows

$$\nabla \cdot \mathbf{u} = 0$$

(4)

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{1}{\rho} \nabla p + \nu \nabla^2 \mathbf{u}$$

(5)

where $\mathbf{u}$, $\nu$ and $\rho$ are the velocity vector, the kinematic viscosity and the pressure, respectively. A pseudo-spectral method is used in the spatial domain to solve the Navier-Stokes equations. Periodic boundary conditions are imposed on the streamwise and spanwise directions to obtain the velocity of the fluid particles. The velocity fields are computed and stored with a time interval $\Delta t = \Delta t_i / \nu = 0.2$ for a period of 3 000. The fluid particle velocity along a particle trajectory is computed by employing a third-order Hermite polynomials in the homogeneous directions and a Chebyshev polynomial in the wall-normal direction. The particles are tracked through the computed velocity fields to obtain the Lagrangian velocity data. The number of particles is of order of $10^6$, which provides a good estimation for the moments of the velocity increments up to $10^9$. In this study, Lagrangian particles are released from three locations with different distances to the wall, i.e., $y^+ = 18.1$, 60.6 and 80.

The Reynolds number considered here is $Re_f = 80$, corresponding to a Taylor-microscale based Reynolds number $Re_{\lambda} \sim 100$. For a DNS simulation, special treatments are required to develop a turbulent flow when the Reynolds number smaller than this value. However, as we will show in this article that despite of the small Reynolds number, the results share the same statistical properties as the high-