Capacity of the Random Multiple Access Method

N. T. Kustov and S. P. Sushchenko

Tomsk State University, Tomsk, Russia

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Abstract—A synchronous conflict model for a group of $K > 1$ stations is designed and used in evaluating the stability of the capacity of the random multiple access method with increasing number of stations and the station individual speed.

1. INTRODUCTION

At present, the multiple access method is extensively used in the design of local area networks (LAN). The well-known approaches to analyzing the capacity of this method [1–5] disregard the number of conflict source in the model. We shall study the capacity of this method in critical cases, when all LAN stations participate in a conflict. We shall design a discrete rivalry model to determine the capacity and probability-time characteristics in terms of the number of stations for a given capacity of a monochannel.

2. SYNCHRONOUS RIVALRY MODEL

2.1. A Group of Homogeneous Sources

We assume that the LAN contains $K$ stations (data sources) and the monochannel seizure time depends on the slot (conflict window) length $t_c$ [5, 6]. All stations are independent and equal, there are always data blocks for transmission and all stations participate in conflicts. The binary exponential delay mechanism is used in the rivalry procedure [6]. Delay is a multiple of the slot size $t_c$ and rivalry of LAN stations continues until the monochannel is seized by a station and successful transmission. In retransmissions, stations, upon detecting a conflict between other stations, continue the rivalry in succeeding retransmissions according to the delay procedure. Retransmissions are not restricted in number.

To simplify analysis, we assume that, irrespective of the number of LAN stations involved in a conflict, the retransmission counters of all sources are increased and reset after a successful data transmission by one of the rival stations. This assumption idealizes the mechanism of variation of retransmission counters of rival stations and is helpful in analytically studying the random multiple access method.

Let $t$ be the retransmission delay at a LAN station, let $n$ (measured in intervals $t_c$) be the time prior to startup of retransmission by the first station of a group of $K$ stations in a routine conflict resolution cycle. Since the retransmission startup instants are multiples of conflict window length, the station that had started transmission first (at instant $t = n$) captures the monochannel if the random delay of other stations is $t > n$. The conditional probability of the $N$th retransmission after the $(N - 1)$th conflict between $K > 1$ stations is the sum of the products of the probabilities $p(t = n)$ that the delays of $k > 2$ stations are equal and the probabilities $p(t > n)$ that the delay is greater than the retransmission startup instant of the remaining $K - k$ stations:

$$
\pi(N, K) = \sum_{k=2}^{K} \left( \begin{array}{c} K \\ k \end{array} \right) \sum_{n=0}^{S_N} p_k^n(t = n)p_{K-k}^{S_N}(t > n),
$$

(1)
where $S_N$ is the upper bound of the delay of the $N$th retransmission. Obviously, the conditional probability of the conflict-free $N$th retransmission is the inverse of the conditional probability of conflict: $P(N, K) = 1 - \pi(N, K)$.

The mean conditional time up to a conflict and success in the $N$th retransmission expressed in $t_c$ are

$$t(N, K) = \frac{1}{\pi(N, K)} \sum_{k=2}^{K} \binom{K}{k} {S_N \sum_{n=1}^{N} np^k(t = n)p^{K-k}(t > n)},$$

$$\tau(N, K) = \frac{K}{P(N, K)} \sum_{n=1}^{S_N} np(t = n)p^{K-1}(t > n).$$

Since $K \geq 2$ stations run into conflict in the first transmission due to the presence data blocks for transmission in them, the conflict resolution time is distributed according to the geometric law with parameter $P(N, K)$ (dependent on the retransmission number $N$) and cycle time $t(N, K)$ between retransmissions. The probability function of rivalry duration is

$$f(N, K) = P(N, K) \prod_{n=1}^{N-1} \pi(n, K),$$

and the distribution function of conflict resolution time is

$$F(N, K) = \sum_{n=1}^{N} f(n, K).$$

According to the random multiple access standard [6] for conflict resolution based on an optimal decentralized control strategy for random access to multiple-user data transmission medium, retransmission schedule is uniformly distributed in the range $[0, 2^N - 1]$. Therefore, $S_N = 2^N - 1$, $p(t = n) = 1/2^N$, and relations (1), (2), and (3) can be rewritten as

$$\pi(N, K) = 2^{-NK} \sum_{k=2}^{K} \binom{K}{k} \sum_{n=0}^{2^N-1} [2^N - 1 - n]^{K-k},$$

$$= 2^{-NK} \sum_{k=2}^{K} \frac{(-1)^{K-k}}{k} \binom{K}{k-1} \left\{ B_{K-k+1} - B_{K-k+1}(1 - 2^N) \right\},$$

where $B_m$ is the Bernoulli number and $B_m(x)$ is a Bernoulli polynomial [7], and

$$t(N, K) = \frac{1}{\pi(N, K)2^{NK}} \sum_{k=2}^{K} \binom{K}{k} \sum_{n=1}^{2^N-1} n[2^N - 1 - n]^{K-k},$$

$$\tau(N, K) = \frac{K}{P(N, K)2^{NK}} \sum_{n=1}^{2^N-1} n[2^N - 1 - n]^{K-1}.$$

The mean time of transmission of a frame from a station (mean conflict resolution time) is the sum of times of first transmission in which stations run into a conflict, conflicts, and successful transmission

$$T(K) = \sum_{N=1}^{\infty} \left\{ t_M + t_c + t_c \sum_{n=1}^{N-1} [t(n, K) + 1] + t_c \tau(N, K) + t_K \right\} f(N, K)$$

$$= t_M + t_K + t_c \overline{T}(K),$$