From Time Inversion to Nonlinear QED

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In Minkowski flat space-time, it is perceived that time inversion is unitary rather than antiunitary, with energy being a time vector changing sign under time inversion. The Dirac equation, in the case of electromagnetic interaction, is not invariant under unitary time inversion, giving rise to a "Klein paradox." To render unitary time inversion invariance, a nonlinear wave equation is constructed, in which the "Klein paradox" disappears. In the case of Coulomb interaction, the revised nonlinear equation can be linearized to give energy solutions for Hydrogen-like ions without singularity when nuclear number $Z > 137$, showing a reversed energy order pending for experimental tests such as Zeeman effects. In non-relativistic limit, this nonlinear equation reduces to nonlinear Schrödinger equation with soliton-like solutions. Moreover, particle conjugation and electron-proton scattering with a nonsingular current-potential interaction are discussed. Finally the explicit form of gauge function is found, the uniqueness of Lorentz gauge is proven and the Lagrangian density of quantum electrodynamics (QED) is revised as well. The implementation of unitary time inversion leads to the ultimate derivation of nonlinear QED.

1. INTRODUCTION

Decades ago, Wigner\(^{11}\) first pointed out the significance of time inversion and later made detailed discussions on antiunitary time inversion (Wigner\(^{2}\)). His theory about time inversion is based on a classical motion picture (Wigner\(^{3}\)): "Time inversion ... replaces every velocity by the opposite velocity, so that the position of particles at $+t$ becomes the same as it was, without time inversion at $-t$ ..."

Also well known is Einstein’s relativity theory (Einstein\(^4\)) that completely changes our perception of space and time, as best manifested by

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Minkowski’s “world-postulate” (see Minkowski’s paper in Ref. 4). Minkowski unifies space and time by introducing time as an independent coordinate in addition to three space coordinates. Usually “Minkowski space-time” refers to four-dimensional flat space-time in special relativity, in which relativistic quantum mechanics and quantum electrodynamics (QED) are established.

On the development of quantum mechanics and relativity, there always exist heated debates on the physical meaning of the theories. Dirac however was indifferent in such debates. Instead he had been looking for mathematical possibilities to reconcile quantum mechanics and relativity. As a consequence, he founded relativistic quantum mechanics with a relativistic wave equation called “Dirac equation” (Dirac\(^{(5)}\)). However, the Dirac equation was also not perfect. In the following year, Klein\(^{(6)}\) found a paradox in the Dirac equation. Now seven decades has gone by, the “Klein paradox” has still not been resolved mathematically, though it can be explained away by physical reasoning.

This paper is organized as follows. Given that a correct concept in Newtonian classical mechanics may not be necessarily correct in Einsteinian relativistic mechanics, first I am going to show that the above mentioned classical motion picture is just one of those concepts, correct classically but not relativistically. Then I will proceed to clarify what is supposed to mean to make a time inversion in special relativity. Based on the general principles of quantum mechanics and special relativity, it can be shown that time inversion turns out to be a unitary transformation with energy being a time vector changing sign under time inversion in Minkowski space-time.

With unitary time inversion understood, the next step is to study the transformation of the Dirac equation. For clarity, let us name the Dirac equation in the case of electromagnetic interaction as Dirac EM-equation to distinguish it from the Dirac equation in other cases. What happens is: the Dirac EM-equation is not invariant under unitary time inversion. As a direct consequence of this non-invariance, the Dirac EM-equation has a non-symmetric positive-negative energy spectrum with a mathematical singularity that causes a “Klein paradox.” Here in this paper, I do not intend to argue too much about whether or not the “Klein paradox” has physical meaning. Rather I hold such a point of view that we would be better off from the outset without mathematical singularity.

To implement unitary time inversion, I come up with a revised equation that looks similar to the Dirac EM-equation on the one hand, while differs from the Dirac EM-equation in several aspects on the other. The main difference is: the revised equation preserves the invariance under