Vortex Dynamics in Superfluid $^4$He at Very Low Temperatures

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Recently McClintock et al. observed the free decay of a vortex tangle in superfluid $^4$He at mK temperatures. Since the system at such low temperatures is free from normal fluid and usual mutual friction, the mechanism of the free decay is unknown. In order to understand this phenomenon, this work studies numerically the vortex dynamics without the mutual friction. The absence of mutual friction prevents the vortex from smoothing. The resulting kinked structure promotes vortex reconnection, thus making lots of small vortex loops. Such cascade process as breakup of vortices to smaller ones can decay the vortex line density. This paper describes the decay of vortex tangle under the localized induction approximation, and that of four vortex rings under the full nonlocal calculation.

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1. INTRODUCTION

Recently McClintock et al. observed the decay of the vortex tangle in superfluid $^4$He at mK temperatures. The first important point is that the tangle does decay in spite of the absence of normal fluid and mutual friction. The second is that the decay rate becomes independent of the temperature below 70mK. This mechanism is unknown.

The numerical analysis of vortex dynamics, pioneered by Schwarz, gave an excellent picture of the self-sustaining vortex tangle (VT) for superfluid turbulence; VT is sustained by the continuous creation of vortex singularities through the reconnection, and the vortex ballooning and shrinking due to the mutual friction under any applied field. Using the idea of dynamical scaling,
Schwarz obtained the dynamical equation of the vortex line density (VLD) \( L \) which had been written down by Vinen,\(^3\) where the dynamics occurs only as the result of mutual friction. At mK temperatures, however, this dynamics does not work because of the absence of mutual friction. Recently Nore et al. studied numerically the nonlinear Schrödinger equation and found that in VT the incompressible kinetic energy is transferred to the energy of acoustic excitations.\(^5\) However, VLD is found to increase monotonically, and the correspondence with the experimental results is not so clear.

In connection with the experiments by McClintock et al.,\(^1\) we study numerically the vortex dynamics without mutual friction. Our numerical method is very similar to that of Schwarz\(^2\) and described in our previous paper.\(^4\) A quantized vortex filament is represented by the parametric form \( s = s(\xi, t) \), and moves subject to the superflow velocity field given by the Biot-Savart expression. The propagation velocity \( \dot{s} \) of the vortex filament at point \( s \) is divided into two components:

\[
\dot{s} = \frac{\kappa}{4\pi} \mathbf{s}' \times \mathbf{s}'' \ln \left( \frac{2(\ell_+ \ell_-)^{1/2}}{e^{1/4}a_0} \right) + \frac{\kappa}{4\pi} \int' \left( \mathbf{s}_1 - \mathbf{s} \right) \times \frac{d\mathbf{s}_1}{|\mathbf{s}_1 - \mathbf{s}|^3},
\]

where \( \kappa \) is the quantized circulation and \( a_0 \) is the vortex core radius. The first term indicates the local-induced field due to a curved line element acting on itself, where \( \ell_+ \) and \( \ell_- \) are the lengths of the two adjacent line elements that hold the point \( s \) between, and the prime denotes differentiation with respect to the arc length \( \xi \). The second term represents the nonlocal field made by the line elements except for the neighborhood of \( s \). When two vortices approach within a critical distance, they are assumed to reconnect.\(^2\)

2. LIA Calculation

First we made the calculation with the localized induction approximation (LIA) that neglects the nonlocal term, which succeeded in the dynamics of VT at finite temperatures.\(^2\) Fig.1(a) shows VT made under an applied flow and the mutual friction at \( T = 1.6 \text{K} \). Switching off the applied flow and the mutual friction changes this VT to that of Fig.1(b) in a short time. The absence of mutual friction makes VT very kinked, which is the important point in this vortex dynamics. If the system is at a finite temperature, the mutual friction tends to reduce the length of the vortex line whose local curvature exceeds a critical value dependent on the applied flow.\(^5\) As a result, the damping of kinks or bumps causes the vortex line to become smooth. However, the system at a very low temperature is free from this mechanism, and cannot prevent the vortex lines from becoming more and more kinked. Then lots of small vortex loops appear through the reconnection of such