LEARNING TO COORDINATE: A RECURSION THEORETIC PERSPECTIVE

ABSTRACT. We consider two players each of whom attempts to predict the behavior of the other, using no more than the history of earlier predictions. Behaviors are limited to a pair of options, conventionally denoted by 0, 1. Such players face the problem of learning to coordinate choices. The present paper formulates their situation recursion theoretically, and investigates the prospects for success. A pair of players build up a matrix with two rows and infinitely many columns, and are said to “learn” each other if cofinitely many of the columns show the same number in both rows (either 0 or 1). Among other results we prove that there are two collections of players that force all other players to choose their camp. Each collection is composed of players that learn everyone else in the same collection, but no player that learns all members of one collection learns any member of the other.

1. INTRODUCTION

Sam and Sally like to meet daily in the park, pretending each time that it’s yet another chance encounter, walking side by side in shy silence. Each shows up punctually at either noon or 6:00 p.m. hoping the other will have made the same choice. The shifting constraints on their schedules, however, make it hard to predict who will select which time of arrival, and both suffer disappointment when there is mismatch. So both Sam and Sally set about trying to predict the other’s choices, desiring to act in concert. Their predictions are based on no more than the history of earlier events. For example, on the sixth morning each might contemplate the matrix:

<table>
<thead>
<tr>
<th></th>
<th>noon</th>
<th>6:00</th>
<th>noon</th>
<th>noon</th>
<th>6:00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sam:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sally:</td>
<td>noon</td>
<td>noon</td>
<td>6:00</td>
<td>6:00</td>
<td>6:00</td>
</tr>
</tbody>
</table>

Their separate decisions extend the matrix to a sixth column, which helps determine the choices made on the seventh morning, and 50 on without end. We can think of Sam’s policy as a function that maps each such matrix (of any finite size) into the set \{noon, 6:00 p.m.\}, and similarly for Sally.

It is said that Sam “learns” Sally’s policy – for short, “Sam learns Sally” – just in case he eventually begins to select arrival times that match Sally’s forever after. (In such circumstances, we may also say that Sally learns Sam.) If Sam is clever enough, Sally could embody any of a wide range of policies without compromising ultimate success, and in this case we say that Sam learns the entire set of potential policies, even though only one of them will be embodied by Sally’s dispositions.

Sam and Sally face the problem of learning to choices. The present paper formulates their situation abstractly and investigates the prospects for success. To keep matters simple, we consider only two players facing the same two options on each trial; the options are denoted 0 and 1. A player will thus be identified with a function from the set of all finite binary sequences into \{0, 1\}, where any such sequence is conceived as the history of moves of an opposing player. From a sequence of length \(n\), a player can reconstruct the \(2 \times n\) binary matrix that includes his own responses through move \(n\). So it is not necessary to represent both rows of the matrix explicitly in players’ inputs; just the opposing player’s moves suffice. In the obvious way, a pair of players build up a matrix with two rows and infinitely many columns. The players are said to “learn” each other if the rows in cofinitely many of the columns agree (that is, both are 0 or both are 1). This conception of players and coordination is threadbare, but highlights the cognitive problem raised by repeated games between the same participants; each must discover a strategy that fits the other’s play. A similar paradigm of learned coordination is raised in (Kelly 1996, pp. 267–8).

The foregoing paradigm will be cast in a recursion-theoretical framework, similar to the development of Formal Learning Theory (Jain et al., 1999). Within the latter tradition (and also in the paradigm to be developed here), the hypotheses of the learner are generated by a computational process that meets various constraints but need not be justified by recourse to probability. Discussion of the contrast between Formal Learning Theory and probabilistic approaches to induction is available in (Earman, 1992; Kelly, 1996; Martin and Osherson, 1998) and in references cited there.

After presentation of the coordination paradigm in the section, we discuss some of its properties, including its relation to Formal Learning Theory. One of our theorems concerns the existence of two collections of players that force all other players to choose their camp. Each collection is composed of players that learn everyone else in the same collection. But no player that learns all members of one collection learns any member of the other. Other results concern cooperation by special classes of players, for