ABSTRACT. In this note we introduce the notion of K-player additive extension of a symmetric two-player game and prove a result relating the equilibria in mixed strategies in the two games. Then we apply the result to the Borda electoral competition game.

KEY WORDS: Two-player, Borda electoral competition

1. INTRODUCTION

Finite two-player zero-sum games have occupied game theorists for a long time and a lot of results have been established for this family of games. In terms of applications to social sciences the zero-sum framework is perfectly suited to deal with situations of extreme competition, i.e., situations where there is no place for cooperation between the players. A good example of such application in the two-person case is the classical ‘Downsian’ model of electoral competition between two parties. Precisely two political parties compete for the votes of an electorate through the policy positions they adopt in a set $X$. Different payoff functions can be considered depending on the objectives pursued by the parties but in all the versions traditionally considered the objectives are assumed to be similar and therefore the game, in addition to be zero-sum is also symmetric. In their analysis of these games Laffond, Laslier and Le Breton (1993, 1997) have derived general new results concerning the uniqueness of equilibrium in mixed strategies for the class of finite symmetric two-player zero-sum games.

The analysis of multiparty electoral competition is a far more complicated subject as reflected by Shepsle (1991). In its present development the theory deals with the case where the set of policy
positions $X$ is the interval $[0,1]$ and to the best of our knowledge no general results have been established in the finite framework. Multiparty electoral competition games belong to the general family of $K$-player symmetric zero-sum games where $K$ denotes the number of parties in competition. The purpose of this note is to provide a first step in that direction. Precisely we will analyse the Borda electoral competition game (Shepsle 1991), i.e., the game where each party maximises the Borda score it gets through the policy position it adopts in $X$. To perform this task we introduce the notion of $K$-player additive extension of a symmetric two-player zero-sum game which is a notion having an independent intrinsic interest. Our main result in Section 2 states that if the original symmetric two-player zero-sum game has a unique equilibrium in mixed strategies $(x, x)$ then its $K$-player additive extension has also a unique equilibrium in mixed strategies $(x, x, \ldots, x)$. In Section 3 we show how to use this general result to characterize the ‘equilibrium’ of the Borda electoral competition game by using results from Laffond, Laslier and Le Breton (1997).

2. NOTATIONS, DEFINITIONS AND RESULT

A $K$-player normal form game is a $2K$-tuple $\{(S_k, G_k)\}_{1 \leq k \leq K}$ where for all $k \in \{1, \ldots, K\}$, $S_k$ is a set and and $G_k$ is a function from $S = \prod_{k=1}^{K} S_k$ into $\mathbb{R}$ denoting respectively the set of pure strategies and the payoff function of player $k$. It is symmetric if $S_k = S_{k'} \equiv X$ for all $k, k' \in \{1, \ldots, K\}$ and for all permutation $\sigma$ on $\{1, \ldots, K\}$, all $k \in \{1, \ldots, K\}$ and all $s \in S$, $G_k(s^\sigma) = G_{\sigma(k)}(s)$ where $s^\sigma = (s_{\sigma(1)}, \ldots, s_{\sigma(K)})$. It is finite if for all $k \in \{1, \ldots, K\}$, $S_k$ is finite. It is constant sum if there exists a constant $C$ such that for all $s \in S$, $\sum_{k=1}^{K} G_k(s) = C$. It is zero-sum if $C = 0$.

From now we will consider only finite games. The mixed extension of a finite $K$-player normal form game $\{(S_k, G_k)\}_{1 \leq k \leq K}$ is the $K$-player normal form game $\{(P_k, M_k)\}_{1 \leq k \leq K}$ where for all $k \in \{1, \ldots, K\}$, $P_k$ is the set of probability distributions over $S_k$.