Of the various accounts of negation that have been offered by logicians in the history of Western logic, that of negation as cancellation is a very distinctive one, quite different from the explosive accounts of modern “classical” and intuitionist logics, and from the accounts offered in standard relevant and paraconsistent logics. Despite its ancient origin, however, a precise understanding of the notion is still wanting. The first half of this paper offers one. Both conceptually and historically, the account of negation as cancellation is intimately connected with connexivist principles such as \( \neg (\alpha \to \neg \alpha) \). Despite this, standard connexivist logics incorporate quite different accounts of negation. The second half of the paper shows how the cancellation account of negation of the first part gives rise to a semantics for a simple connexivist logic.

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1. Negation as cancellation

1.1. Three accounts of negation

Accounts of negation fall into three distinct kinds, which can be defined in terms of the behavior of the contradiction, \( \alpha \land \neg \alpha \), to which the negation gives rise.\(^1\)

The first account is a total one. According to this, the content of the contradiction \( \alpha \land \neg \alpha \) is total. A contradiction therefore entails everything, entailment being cashed out in terms of containment of content. Accounts of this kind are, historically, the most recent of the three kinds, but are also now the most orthodox, the position being entrenched in modern explosive logics such as “classical” logic and intuitionistic logic.

The second account is a partial account. According to this, a contradiction \( \alpha \land \neg \alpha \) has, in general, some content, but not all. A contradiction therefore entails some things, but not others. Such accounts are now familiar from modern paraconsistent logics. For example, in standard relevant logics \( \alpha \land \neg \alpha \) entails \( \alpha \) and \( \neg \alpha \), but not an arbitrary \( \beta \).

The third kind of account is the least familiar to modern logicians, though it is, arguably, the most venerable. It is the null account. According to such an account, a contradiction has no content. Accordingly, \( \alpha \land \neg \alpha \) entails nothing. Consider the following quotation from Strawson (1952, p. 2f.):

Suppose a man sets out to walk to a certain place; but when he gets half way there, he turns round and comes back again. This may not be pointless. But, from the point of view of change of position, it is as if he had never set out. And so a man who contradicts himself may have succeeded in exercising his vocal chords. But from the point of view of imparting information, or communicating facts (or falsehoods) it is as if he had never opened his mouth. He utters words, but does not say anything. . . The point is that the standard function of speech, the intention to communicate something, is frustrated by self-contradiction. Contradiction is like writing something down and erasing it, or putting a line through it. A contradiction cancels itself and leaves nothing.

The quotation illustrates a null account very clearly. The difference between such an account and a total account is also clear. On a total account, a person who asserts
a contradiction is most certainly in a different position from one who has not opened their mouth. They are committed to everything, not nothing. They may also have communicated a falsehood – if they can get their hearer to believe them.

Despite this, the formal account of negation that Strawson gives later in the book is the familiar explosive one of classical logic. This illustrates a further fact: explosive accounts of negation are so entrenched in some people’s minds that even when they give an entirely different account, this fact is opaque to them. Both themes are further illustrated in the following quotation from Findlay:²

... a contradiction is for the majority of logical thinkers, a self-nullifying utterance, one that puts forward an assertion and then takes it back in the same breath, and so really says nothing. It can readily be shown that a language system which admits even one contradiction among its sentences, is also a system in which anything whatever can be proved. . . .

1.2. A little history

The cancellation view of negation is, as I said, the oldest of the three kinds of view of negation. It’s germ is to be found in Aristotle. At Prior Analytics 57ª³, Aristotle claims that contradictories cannot both entail the same thing. Now suppose that (in modern notation) \( \alpha \land \neg \alpha \) entailed both \( \alpha \) and \( \neg \alpha \), then, by contraposition (which Aristotle endorses immediately before this), each of \( \neg \alpha \) and \( \neg \neg \alpha \) would entail \( \neg (\alpha \land \neg \alpha) \). Hence, as must have been obvious to Aristotle, a contradiction cannot entail both conjuncts, and so, presumably, either conjunct. These are the prime candidates for something a contradiction may entail.³

Presumably influenced by Aristotle, Boethius certainly seems to have subscribed to a null account of negation.⁴ More of him anon. The view must have been a commonplace in the early middle ages since Abelard tells us:⁵

No one doubts that [a statement entailing its negation] is improper and embarrassing (inconveniens) since the truth of one of two propositions which divide truth [i.e., contradictories] not only does not require the truth of the other but rather entirely expels and extinguishes it.

The view certainly became less than orthodox in the later middle ages, for reasons that we will come to, but it survives in places well into the 18th century. For example, Berkeley tells us in the Analyst:⁶

Nothing is plainer than that no just conclusion can be directly drawn from two inconsistent premises. You may indeed suppose any thing possible: But afterwards you may not suppose anything that destroys what you first supposed: or, if you do, you must begin de novo. . . . [When] you . . . destroy one supposition by another . . . you may not retain the consequences, or any part of the consequences, of the first supposition so destroyed.

And despite the modern dominance of explosive views of negation, the cancellation view still resonates into the 20th century, as we have already seen.

1.3. The account made precise

Consequence relations incorporating total or partial accounts of negation are now very familiar. The same cannot be said of null accounts, however, despite the antiquity of the tradition. In this section I will suggest one. It is clear that a null account of negation gives rise to a paraconsistent logic. Most such logics are partial, not null, however. The following is an exception.

We will stick to propositional logic with the connectives \( \neg, \land \) and \( \lor (\lor) \) can be defined in the usual way. The extension of the idea to quantifier logic is obvious. I will use lower case Roman letters for propositional parameters, lower case Greeks for arbitrary formulas, and upper case Greeks for sets of formulas. I will often omit set braces. I will use “consistent” and its cognates for classical consistency, and use \( \models \) for classical entailment.

The main problem in formulating a null account of negation, as should be clear, is how to make sense of the idea that a contradiction has no content. We will enforce this in the most simple-minded way. Let us say that:

\[ \Sigma \models \alpha \text{ iff } \Sigma \text{ is consistent, and } \Sigma \models \alpha \]

Thus, an inconsistent set of sentences entails nothing.⁷ A feature of this definition is that it does not validate contraposition. As is easy to check, \( p \not\models p \lor \neg p \), but \( \neg(p \lor \neg p) \not\models \neg p \). For reasons that will become clear later, it will be useful to have an account with contraposition built into it. An alternative such definition is:

\[ \Sigma \models \alpha \text{ iff } \Sigma \text{ is consistent, } \neg \alpha \text{ is consistent and } \Sigma \models \alpha \]

I will call this account the symmetrised account. In what follows, \( \models \) may be either of the above relations unless otherwise stated.