A Dugdale–Barenblatt model for a plane stress semi-infinite crack under mixed mode concentrated forces

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Abstract. In this paper, a mixed mode Dugdale–Barenblatt model has been established for a semi-infinite crack in an ideally elastic-plastic thin plate loaded by a pair of self-equilibrating concentrated forces at the crack lips. Cohesive forces are introduced into a plastic strip in the elastic body. By superposing the two linear elastic fields, one evaluated with the external loads and the other with the cohesive forces, the problem is treated in Dugdale–Barenblatt’s manner. The physical domain is mapped with a complementary domain of the unit circle by using the Schwartz–Cristoffel transformation. The Muskhelishvili complex potentials are used to find out the stress-intensity factors due to the two separated fields. The analytical approach leads to establish a few transcendence equations from which the quantities of interest, such as the direction and the length of the plastic strip, the crack opening distance etc., can easily be deduced by standard numerical methods.

Key words: Fracture, elastic-plastic crack, Dugdale–Barenblatt model, mixed mode crack.

1. Introduction

The initiation and the continuation of crack growth in elastic-plastic materials are subjects of numerous studies. Since the elastic-plastic analysis of cracked structures is very complicated, establishing physical models is an efficient way to simplify the mathematical analysis and to obtain significant results. Without entering into the detail of the development of individual voids or micro-cracks in the vicinity of the crack tip, we are essentially interested in the continuum modeling through constitutive assumptions about stresses and strains. In this topic, several crack models were established by different authors. Barenblatt (1959) first suggested a method of utilizing the model of an elastic body to incorporate cohesive forces into the theory of cracks. It is assumed that the crack surfaces start to separate if some measures of the cohesive stresses exceed a critical level. According to this model, all the components of stress at an arbitrary point near the crack tip must be finite. Based on the experimental observations, Dugdale (1960) studied the mode I cracks in thin foils of soft steel by using the cohesive zone model. Bui and Ehrlacher (1981) introduced the softening behavior into the constitutive laws and established a damage accumulation model. The micro-damage is assumed to be describable by some internal state variables. Also in this model nonsingular stress and strain states may result and the crack growth condition can be formulated directly in terms of stress or strains. More recently, Rice (1992) carried out an analysis of dislocation formulation at an elastic-plastic crack tip by using the Peierls (1940) concept. A periodic relation is assumed to hold between shear stress and sliding displacement along a crystal slip plane emanating from a crack tip. Other models can also be found in previous literature (Eringen et al., 1977 etc.).
Among these models, the Dugdale–Barenblatt model (hereafter refereed as the D–B model) is the ‘oldest’ but efficient one for investigations of plane-stress cracked structures. In this model, the cohesive forces are assumed to be constantly distributed at the ‘end’ region of the crack in which the opposing crack faces are closed to each other and formed a plastic strip. The mathematical development is simplified with this assumption. Most mode I loaded cracks can be studied by using this model. The experimental results showed satisfactory agreement with the theoretical model (Dugdale, 1960). However, the mixed mode elastic-plastic cracks, which can be found in many real structures, have not been studied thoroughly by using the D–B model so far. We believe that establishing models of D–B type for mixed mode cracks and deducing some simple results will be useful in engineering applications.

In this paper, we present a D–B model for a semi-infinite plane-stress crack under self-equilibrating concentrated forces applied on the crack lips. The material is supposed to have an ideally elastic-plastic behavior. The plastic strip where applied the cohesive forces is supposed to be a straight line and may be oriented differently from the crack axis. Similarly to Dugdale (1960), we can suppose that in thin foils, the plasticity develops at the direction where the shear stresses reach their maximum values, at an angle of 45° to the structure plane. Therefore, the plastic strip will be perpendicular to the direction of the maximum tensile stresses ahead of the crack tip. At the end of the plastic strip, the stresses must be finite according to Barenblatt and Cherepanov (1961). Under these conditions, an analytical approach has been carried out to establish the corresponding D–B model.

A conformal mapping is first accomplished to transform an infinite plane containing a semi-infinite crack with a branched arm into a complementary region of a unit circle. Following the D–B approach, the problem can then be resolved by superposition of two linear elastic fields, a field evaluated neglecting cohesive forces and one corresponding to the action of forces of cohesion and no others. The Muskhelishvili (1958) complex potentials are used to find out the stress-intensity factors of the two separated fields. By superposing the two fields with the hypothesis mentioned above, all quantities of interest, especially the angle and the length of the plastic strip, the crack opening distance and the $J$-integral of Rice (1968), are deduced.

2. Superposition schema and the conformal mapping

Let us consider a semi-infinite crack in a thin plate placed along the negative $x$-axis and opened by concentrated self-equilibrating forces $P$ and $Q$ applied to the crack lips at a distance $c$ away from the crack tip. The material is supposed to have an ideally elastic-plastic stress-stain response. Similarly to the D–B model, one can suppose that the plasticity develops along a straight thin layer in material ahead of the crack tip and oriented at an angle $m\pi$ with respect to the crack axis (Figure 1a). This problem can then be resolved by superposing the two linear elastic stress field. The first field is the elastic solution of a branched crack under the concentrated self-equilibrating forces $P$ and $Q$ (Figure 1b). The second field is that of the same branched crack closed by the cohesive forces applied along the branched crack lips (Figure 1c). The values of the stresses at the new crack tip must be finite, the total stress-intensity factors $K_I$ and $K_{II}$ due to the two elastic fields will be zero. According to this condition, the branched angle $m\pi$, the length of the plastic zone $s$ and other quantities of interest can be found out.