A micromechanical model for a viscoelastic cohesive zone

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Received 18 November 1999; accepted in revised form 6 July 2000

Abstract. A micromechanical model for a viscoelastic cohesive zone is formulated herein. Care has been taken in the construction of a physically-based continuum mechanics model of the damaged region ahead of the crack tip. The homogenization of the cohesive forces encountered in this region results in a damage dependent traction-displacement law which is both single integral and internal variable-type. An incrementalized form of this traction-displacement law has been integrated numerically and placed within an implicit finite element program designed to predict crack propagation in viscoelastic media. This research concludes with several example problems on the response of this model for various displacement boundary conditions.

Key words: Fracture, interfaces, cohesive elements, viscoelasticity.

Introduction

Fracture mechanics is one of the most important contributions of applied mechanics in the twentieth century. Starting with the seminal paper of Griffith [1] in 1920, the field of fracture mechanics has flourished under the assumption that crack growth occurs whenever the energy release rate exceeds the critical energy release rate, which is regarded as a material property. This concept has resulted in important advances in engineering and science over the last quarter of a century. Yet there still remain issues that have not been sorted out, such as subcritical crack propagation [2–6] and apparent crack tip (R-)toughening [7]. One method of accounting for these seemingly enigmatic problems that has come under recently increasing scrutiny is the approach of implementing a cohesive zone at the crack tip, following the early works of Dugdale [8] and Barenblatt [9]. In this method, a zone of non-zero tractions is placed at the crack tip, often thereby removing the stress singularity. Since the list of researchers that have studied this approach is much too long to detail here, we mention only those that have dealt with cohesive zones models in viscoelastic media [10–15] . In addition, a number of researchers have begun to use this approach in computational models for predicting crack propagation [16–22]. A particularly inviting aspect of this approach is that, depending on the constitutive model used for the traction-displacement relationship along the crack faces, history dependence can be incorporated into the critical energy release rate as a predictive capability in fatigue models [23]. Unfortunately, the choice of a constitutive model for the cohesive zone is the weakest part of this approach to the fracture mechanics problem. Due to the small scale of the actual cohesive zone in most materials, it is experimentally quite difficult to determine the precise nature of the constitutive behavior in the cohesive zone. Thus, it has become commonplace to postulate a phenomenological form for the cohesive zone constitutive model that contains one or more free parameters. While this approach has been shown to qualitatively predict some phenomena that have not been explained by analyses
involving singular stress fields and/or constant critical energy release rates, the inability to construct physically based cohesive zone models has impeded this approach.

In this paper, a methodology is reviewed for constructing a micromechanically based constitutive model for cohesive zones. The paper begins with a description of the global boundary value problem describing the crack of interest, followed by a brief review of the mechanics of the damaged zone. This is then followed by the solution of a micromechanics problem representing the cohesive zone and the subsequent application of a homogenization technique necessary for the construction of a mechanically averaged cohesive zone constitutive model. Some example problems are solved to demonstrate the efficacy of the model.

Problem formulation

The physical behavior of the continuum in the vicinity of a crack tip in polymers can be extremely complex. Local void formation creates a region of material heterogeneity where load-bearing strands of polymeric material called fibrils bridge the distance between the faces of the damaged region. Excellent examples of these fibrillated regions can be found in the work reported by Kramer and Berger [24]. This region, appropriately named a damaged zone, has a constitution which differs greatly from that of the surrounding bulk media. A common observation that has been made regarding damaged zones is that the thickness of this region is very small as compared to the other characteristic dimensions of the region. Therefore when formulating analytic solutions to problems which involve damaged zones, this observation leads to the reasonable assumption that the damaged zone can be approximated by a zone of cohesive tractions which has zero thickness in the undeformed configuration. This mechanically equivalent two-dimensional surface is termed a cohesive zone.

When posing the global boundary value problem for a general body containing cracks, cohesive zones will serve to replace the damaged zones which exist in front of the crack tips. To aide in the construction of the micromechanically-based damage model, a more detailed description of the damaged zone will be examined. A representative volume element (RVE) will be extracted from the damaged zone, and the homogenization of the cohesive tractions in the RVE will result in a governing cohesive zone traction-displacement law with an internal damage parameter.

GLOBAL BOUNDARY VALUE PROBLEM

Consider a general body containing one or more discrete cracks with cohesive zones as shown in Figure 1. The body of interest has an interior $\Omega$, and a boundary $\partial \Omega$ which is comprised of two parts: $\partial \Omega_x$, which denotes the part of the boundary without cohesive zones; and $\partial \Omega_c$, which denotes the part of the boundary with cohesive zones. The variables of state for this analysis are the displacement vector $u_{i}(x_k, t)$, the stress tensor $\sigma_{ij}(x_k, t)$, and the strain tensor $\varepsilon_{ij}(x_k, t)$.

In the absence of body forces and inertial effects, the conservation of linear momentum may be expressed by

$$\sigma_{ji,j} = 0 \quad \text{in} \quad \Omega. \quad (1)$$

By neglecting any body moments, the conservation of angular momentum implies that the stress tensor must be symmetric, i.e.,