Risk Premiums and Benefit Measures for Generalized-Expected-Utility Theories

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Abstract

Standard tools for the analysis of economic problems involving uncertainty, including risk premiums, certainty equivalents and the notions of absolute and relative risk aversion, are developed without making specific assumptions on functional form beyond the basic requirements of monotonicity, transitivity, continuity, and the presumption that individuals prefer certainty to risk. Individuals are not required to display probabilistic sophistication. The approach relies on the distance and benefit functions to characterize preferences relative to a given state-contingent vector of outcomes. The distance and benefit functions are used to derive absolute and relative risk premiums and to characterize preferences exhibiting constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA). A generalization of the notion of Schur-concavity is presented. If preferences are generalized Schur concave, the absolute and relative risk premiums are generalized Schur convex, and the certainty equivalents are generalized Schur concave.

Key words: Preferences, risk, distance function, benefit function, generalized expected utility

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1. Introduction

Over the past fifteen years, the theory of choice under uncertainty has undergone radical change. The pivotal contribution was Machina’s (1982) demonstration that a large class of preferences could be locally approximated by expected-utility functionals and that global preferences inherited properties, such as risk aversion, of the local utility functions. Less progress has been made, however, in developing tools relating to non-local properties of preferences such as the absolute and relative risk premiums used in expected-utility theory.

During this same period, however, the literature on choice under certainty made substantial progress in developing new techniques for characterizing preferences and technologies using the concepts of distance (Färe, 1988) and benefit functions (Luenberger, 1992). In particular, Luenberger (1992, 1994) introduced the benefit function and demonstrated its usefulness in characterizing preferences and Pareto-efficient outcomes. It is
natural, therefore, to ask whether these techniques can be informatively applied to problems of choice under uncertainty.

This paper shows that a wide range of standard tools for the analysis of economic problems involving uncertainty, including risk premiums, certainty equivalents and the notions of absolute and relative risk aversion, can be developed and applied without making specific assumptions on functional form beyond the basic requirements of monotonicity, transitivity, continuity, and the presumption that individuals prefer certainty to risk. In particular, individuals are not required to display probabilistic sophistication, in the sense of Machina and Schmeidler (1992). Our approach relies on the distance and benefit functions to characterize preferences relative to a given state-contingent vector of outcomes, and then derives results directly from the properties of these functions.

After introducing our notation, we start by defining a concept of risk aversion from which mean values and a generalized concept of subjective probabilities can be derived. The distance and benefit functions are then used to derive absolute and relative risk premiums and to derive conditions under which preferences display constant absolute risk aversion (CARA) and constant relative risk aversion (CRRA). An immediate by-product of this discussion is a result characterizing preferences displaying both CARA and CRRA. This result is then used to suggest several flexible functional specifications of preferences satisfying both properties. Finally, a generalization of the notion of Schur-concavity, following Chew and Mao (1995), is presented. It is shown that if preferences are generalized Schur concave, the absolute and relative risk premiums are generalized Schur convex, and the certainty equivalents are generalized Schur concave.

2. Notation

We adopt a state-contingent approach, as in the work of Hirshliefer (1965) and Yaari (1969). That is, we are concerned with preferences over random variables represented as mappings from a state space \( \Omega \) to an outcome space \( Y \subset \mathbb{R} \), or, in the analysis of measures of relative risk aversion, \( Y \subset \mathbb{R}^+ \), where \( \mathbb{R}^+ \) denotes the positive real numbers. Our focus is on the case where \( \Omega \) is a finite set \( \{1, \ldots, S\} \), and the space of random variables is \( Y^S \subset \mathbb{R}^S \). (No major difficulties arise with the extension to general measure spaces.) We make particular use of the unit vector \( 1 \equiv (1, 1, \ldots, 1) \). Preferences over \( Y^S \) are given by a total ordering denoted notationally by \( \prec \).

A preference function is a mapping \( W : Y^S \to \mathbb{R} \) such that \( W(y) > W(y') \) if and only if \( y < y' \). \( W \) is assumed everywhere continuous and nondecreasing. Two alternative function representations of preferences provide the bulwarks of our analysis. The first is a particular version of Luenberger’s (1992) benefit function for the preference structure. The benefit function, \( B : \mathbb{R} \times Y^S \to \mathbb{R} \), is defined for \( g \in \mathbb{R}^S \) by:

\[
B(w,y) = \max \{ \beta \in \mathbb{R} : W(y - \beta g) \geq w \}
\]