An Iterative Method of Point Mapping under Cell Reference for the Global Analysis: Theory and a Multiscale Reference Technique

JUN JIANG and JIAN-XUE XU
Institute of Engineering Mechanics, Xi’an Jiaotong University, Xi’an, Shaanxi 710049, P.R. China

(Received: 24 March 1994; accepted: 16 April 1997)

Abstract. In iterative method of Point Mapping under Cell Reference, a cell co-ordinate system, called cell reference, is built to identify the subregions (cells) in the state space. When the cell reference is equipped with the so-called characteristic functions, it can work as an inspector or a recorder to derive the local dynamics of the subregions from the information provided by the trajectories passing through them. This method can retain the accuracy of the Point Mapping Method but greatly reduce the computational work. In this paper, the theoretic basis for this method is first discussed and a multiscale reference technique is then devised which can select an optimal cell reference and make the method more practicable. Finally, an example for application is presented. It is shown that the present method cannot only accurately and efficiently depict the basins of attraction of a dynamical system but also potentially detect other characteristics of the system.

Key words: Global analysis, cell references, the attracting sets, fractal basin boundary.

1. Introduction

It is well accepted that numerical computation is extremely important in modern nonlinear analysis. In fact, the ready availability of extensive computational resources has led to the current resurgence of interest in nonlinear system behavior, much of which now centers on global system characteristics. Unfortunately, the costs of numerical simulation by the Point Mapping Method are extremely expensive.

To relieve some of above difficulties, Hsu [1] proposed a cell-to-cell mapping theory as well as an unraveling algorithm called Simple Cell Mapping (SCM). Later, Hsu [2] further developed this method to a more complicated one after successfully introducing the theory of Markov chain to the global analysis and denoted it as Generalized Cell Mapping. To retain the continuous state space, an Interpolated Cell Mapping was invented by Tongue and Gu [3]. All of these methods use the approximated mappings instead of point mappings to reduce the computational works. However, they cannot always produce reasonable results, especially for some complex dynamical systems [6].

To retain the accuracy of the Point Mapping Method but enhance its efficiency, a method of Point Mapping under Cell Reference is proposed in [4] by adding a cell co-ordinate system in the state space beside the usual Cartesian one. After introducing the concept of characteristics-preserving distances (although the largest one is not computable, a proposed distance which is several orders greater than the machine accuracy can be always adopted in the computation), the character of a processing trajectory is thought to be determined when it approaches close to a formerly processed trajectory in a distance less than the proposed distance. We termed this method as a preliminary PMUCR. Recently, authors have proposed an iterative method of Point Mapping under Cell Reference [6], or iterative PMUCR in short. In this method, the characteristic functions are defined on the cell reference and the computational process
is divided into an initial step and an iterative step. It is shown that iterative PMUCR, in comparison with the Point Mapping Method, reduces the computational work with a factor $8 \sim 15$, depending on the complexity of the analyzed system.

In Section 2 the theoretic basis for iterative PMUCR is discussed based on the basic properties of attractors and basins of attraction. The method of iterative PMUCR is briefly described in Section 3. In Section 4 a multiscale reference technique is proposed to optimize the cell size of the cell reference. To show the validity of this method, an example of application is presented in Section 5. Finally, we make some conclusions and discussions in Section 6.

2. Theoretic Basis for Iterative PMUCR

2.1. Notations of Motion and Definitions

In analysis a finite dimensional dynamical system is often modeled in form of an ordinary differential equation

$$\dot{x} = F(x, t),$$

(1)

where $x \in D \subseteq \mathbb{R}^N$ is an $N$-dimensional state vector, $t \in \mathbb{R}$ the time variable, and $F$ a vector-valued function of $x$ and $t$. We assume that the components of $F$ are smooth enough to produce at each point $x(x_0, t_0) \in D \subseteq \mathbb{R}$, $x_0 = x(t_0) \in D$, $t_0 \in \mathbb{R}$, a unique solution to (1) called motion of the system:

$$x(x_0, t_0, \cdot) : \mathbb{R} \rightarrow D.$$

If we limit our analysis to the autonomous dynamical systems or the periodic dynamical systems, the dependence of the motion on the starting point of time, $t_0$, is none or not the substantive matter. Since the completely equivalent properties are exhibited on any of the time section for a periodic dynamical system, we will restrict our definitions about periodic systems on a fixed time section. At this time, definitions for autonomous systems and periodic systems are in the same form. The solution of (1) can be simply denoted as

$$x(x_0, \cdot) : \mathbb{R} \rightarrow D.$$

We define a motion curve corresponding to an initial point $x_0 \in D$ as

$$f^t x_0 = x(x_0, R) = \{x(x_0, t) \mid t \in \mathbb{R}\}$$

(2)

and

$$f^{t>0} x_0 = x(x_0, R^+) = \{x(x_0, t) \mid t \in R^+\}$$

(3)

as a positive semi-motion curve. Since a motion on the positive semi-axis of time is of great interest, we will concentrate our analysis on it and call it a trajectory.

If, instead of looking for the continuous time history of a motion of the system, one examines the state of the system at a sequence of discrete instants of time, one is led to the concept of point mapping or Poincaré map. At this time a trajectory represented by (3) could be expressed in the forms

$$f^{n>0} x_0 = x(x_0, Z^+) = \{x(x_0, n\tau) \mid n \in Z^+\},$$

(4)

where $\tau$ is the time interval for sampling the points from a trajectory.