A SYMPLECTIC MAPPING MODEL AS A TOOL TO UNDERSTAND
THE DYNAMICS OF 2/1 RESONANT ASTEROID MOTION

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Abstract. We present a 3-D symplectic mapping model that is valid at the 2:1 mean motion resonance in the asteroid motion in the Sun-Jupiter-asteroid model. This model is used to study the dynamics inside this resonance and several features of the system have been made clear. The introduction of the third dimension, through the inclination of the asteroid orbit, plays an important role in the evolution of the asteroid and the appearance of chaotic motion. Also, the existence of the secondary resonances is clearly shown and their role in the appearance of chaotic motion and the slow diffusion of the elements of the orbit is demonstrated.

Key words: resonance, chaotic motion, diffusion, secondary resonances.

1. Introduction

The explanation of the Kirkwood gaps in the asteroid belt is an old and famous problem in the study of the solar system, which is related to the stability of resonant motion in a nonlinear dynamical system. The interest in this problem was revived after the work of Wisdom (1982,83,85), who showed that the observed gap at the 3:1 mean motion resonance of the asteroid with Jupiter can be explained by purely gravitational forces.


The problem of the Kirkwood gaps, apart from its particular interest in the study of the solar system, is an interesting problem of nonlinear dynamics, related to the stability of resonant motion and the generation of chaos and its long term effect on the evolution of the system. To understand the dynamics of the system, one can consider a hierarchy of perturbations, starting with the simplest model and going gradually to more complicated models, adding more features and consequently more degrees of freedom. The increase of the degrees of freedom introduces new resonances to the system, the secondary resonances and the secular resonances,

which play an important role in the evolution inside the 2:1 resonance. These resonances may overlap and thus form a bridge to connect low and high eccentricity and inclination regions in phase space (Yoshikawa, 1989, Franklin, 1994, Moons and Morbidelli, 1995 and Henrard et al. 1995).

The purpose of this paper is to contribute to the study of the 2:1 resonant asteroid motion in three dimensions, by making use of a suitable symplectic mapping model. The underlying physical system is the Sun-Jupiter-asteroid system, with Jupiter in a fixed orbit. This mapping model can be used to understand the dynamics at this region and guide us on where to focus our attention in numerical simulations. In particular, we address several aspects of the problem as: (a) The effect of introducing the third dimension in the model. (b) The appearance of the secondary resonances and the generation of chaotic motion through their overlap. (c) The effect of the initial phase on the evolution of the system. This mapping model compares well with other models used in the study of the 2:1 resonance, as we shall see in the following.

2. The Mapping Model

The symplectic mapping model that we shall use is based on the averaged Hamiltonian at the 2:1 resonance and corresponds to the elliptic restricted three body problem, in three dimensions, with the Sun and Jupiter as primaries. The planar part of this averaged Hamiltonian has been used in Hadjidemetriou and Lemaitre (1997) and contains all the high eccentricity resonances of the Sun-Jupiter-asteroid model. This was achieved by the introduction of suitable correction terms to the averaged Hamiltonian obtained by the usual perturbation methods. In the present paper we use the averaged Hamiltonian at the 2:1 resonance, in three dimensions, obtained by Šidlichovský (1991). The planar part is the same as that used in the paper by Hadjidemetriou and Lemaitre (1997), and we applied the same correction terms to make the model realistic. We kept first order terms in $e_j$ in the expansion of the averaged Hamiltonian and also first order terms in $\sin^2 \frac{i}{2}$. Since $i'$ is fixed and of the order of $1^\circ$, we ignored all terms of third order in $e$, $\sin \frac{i}{2}$ and $\sin \frac{i'}{2}$.

2.1. The Averaged Hamiltonian

The averaged Hamiltonian is expressed in the resonant action-angle variables (Moons 1997) $S$, $S_z$, $N$, $\sigma$, $\sigma_x$ and $\nu$, given by

\[
S = L - G, \quad S_z = G - H, \quad N = 2L - H,
\sigma = 2\lambda' - \lambda - \varpi, \quad \sigma_x = 2\lambda' - \lambda - \Omega, \quad \nu = -2\lambda' + \lambda + \varpi'.
\]

where

\[
L = \sqrt{(1 - \mu)a}, \quad G = L\sqrt{1 - e^2}, \quad H = G \cos i,
\]

(1)

(2)