Limits of Constructivism: Kant, Piaget and Peirce

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ABSTRACT: The paradox of mathematical knowledge that mathematics cannot be conceived of as completely separated from empirical experience and yet cannot be explained by empiricist epistemology (for a slightly different and more elaborate formulation cf. Blackwell Companion to Epistemology, 2/01), can only be resolved if one accepts that the causal interactions between knower and environment have themselves a generalizing tendency, a sort of continuity, rather than consisting just of singular events. Kant resolves the schism between the continuous and the distinct in a constructivist manner. He assumes that all our knowledge-extending cognitions are synthetic. This synthesis does not lie in the matter of experience but springs from the function of cognizant consciousness. Piaget adhered to a Kantianism where “the categories are not there at the outset”. He conceives of the subject or constructing itself as well as of the changing subject’s structure as the source of the apprehension of the world and believes in a Kantianism which emphasizes man’s active being and potential for unlimited self-development. But he has no use for the Kantian idea of space and time as forms of mathematical intuition.

Kantian thought is also central to Peirce’s philosophy and conception of mathematics. But Peirce emphasizes the role of perception and analysis as its prerequisites. Peirce’s and Piaget’s origins in Kantianism are exhibited when both try to replace the Aristotelian notion of abstraction and generalization by something more suitable for mathematical epistemology. Peirce proposes that “hypostatic abstraction” is the chief explanation for the power of mathematical reasoning and explains: “This operation is performed when something, that one has thought about any subject, is itself made a subject of thought”. Piaget speaks of “reciprocal abstraction” in this context, making it the basis of mathematical knowledge, but separating it completely from empirical abstraction.

I. THE CONTINUITY PRINCIPLE AND THE REALITY OF IDEAL OBJECTS

Reality is infinitely varied compared even with the subllest workings of conceptual or theoretical thought. Reality is not a unique and homogeneous thing. Reality strives for infinite graduation of being, of perspective, and of communication. Therefore, the principle of continuity was the most important methodological and epistemological device since the days of Newton and Leibniz. Lovejoy, in his important The Great Chain of Being, pursues its roots much farther back into Antiquity. Aristotle, although being foremost the great representative of classical logic which rests upon the assumption of the possibility of clear divisions and rigorous classification, had already made the observation that “Nature refuses to conform to our cravings for clear lines of demarcation” and “he first suggested the limitations and dangers of classification” (Lovejoy 1936, 56). Aristotle thereby became responsible for the introduction of the principle of conti-
nake into natural history. 'And the very terms and illustrations used by a hundred later writers down to Locke and Leibniz and beyond, show that they were but repeating Aristotelic's expressions of this idea' (Lovejoy 1936, 58). Even since, these two sides of cognition, the intuitive or figurative and the logical and operative, have remained opposed to one another. The first represents the rich but particular perspective of monadic beings, the second tries to overcome this particularity in an abstractive and formal way. It is on such distinctions between the intuitive and the discursive processes of the mind, writes Peirce, "that the greater systems have been founded". Now the principle of continuity was meant to give the general and the particular their complementary roles. Lovejoy had thus to conclude that the continuity principle was transformed into a temporalized form after its importance disappeared altogether at the beginning of the 19th century and with the appearance of evolutionistic theories on the one hand, which had multiplied since the last quarter of the 18th century, and of constructive structuralism on the other. Kant in particular introduced a radical separation between the reality of things in themselves and of phenomena or appearances, limiting the principle of continuity to the latter. Kant had learnt from Hume that relations are external, that they represent nothing of the essence of the relata. Continuity we find, according to Kant, only in the realm of phenomena as they are synthesized by activity. Kant and Piaget were essentially dualists, starting from the belief that whatever connection, coherence or continuity there is, it is a result of the subject's constructive efforts, rather than a precondition of making these efforts meaningful.

In contrast to Kant's views one should rather understand the continuity principle as stating that we understand something only on the basis of change and evolution, that is from the conditions of its genesis. This formulation, although being rather vague nevertheless runs counter the empiricist interpretation, for instance, John Stuart Mill's interpretation in terms of an assumed "Uniformity of Nature". And it also distinguishes itself from the narrowing of constructivism, which, while taking into account only the activity of the subject as such a condition of genesis, claims that we can only understand what we ourselves have constructed. I believe that in referring to these different interpretations of the continuity principle we may not only obtain a first idea of the differences between Peirce's views on the one side and Piaget's on the other, but are also led to suspect that the notions of space and of perception should provide major points of disagreement for both positions.

The greatest fault of Kant's epistemology, according to Peirce, was at the same time "the greatest merit of his doctrine: it lay in his sharp discrimination of the intuitive and the discursive processes of the mind. ... [Kant] saw far more clearly than any predecessor had done the whole philosophical import of this distinction. This was what emancipated him from Leibnizianism, and at the same time turned him against sensationalism. ... But he drew too hard a line between the operations of observation