Generalized Thermodynamics of Maximum Work in Finite Time*

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(Received: October 20, 1997)

Abstract. We consider thermodynamic behaviour of thermal machines founded on kinetic rather than static origins. Their models, which are formulated for finite time transitions, simplify to models of classical thermodynamics in the limiting case of an infinite duration. An extended exergy is derived as a finite-time extension of the classical thermodynamic work delivered from a system of a body and its environment. With this quantity enhanced bounds can be determined for active continuous and cascade processes, in which there is an indirect energy exchange between two subsystems through the working fluid of an engine, a refrigerator or a heat pump. These bounds refer to systems with finite exchange area or with a finite contact time. An economic framework of this theory is outlined.

For both continuous and discrete processes, nonlinear thermodynamic models are derived from a combination of the energy balance and transfer equations. These models serve as constraints in the problem of work optimization. Variational and optimal control approaches are developed which are analogous to those found in analytical mechanics. Variational calculus is used along with some aspect of the canonical transformation theory to maximize work and discuss the role of a finite process intensity and of a finite duration.

The optimality of a definite irreversible process for a finite-time transition of a controlled fluid is pointed out as well as a connection between the process duration, optimal dissipation and the optimal process intensity measured in terms of a hamiltonian, a dissipative quantity. It is shown that limits of the classical availability theory should be replaced by stronger limits which are obtained for finite time processes, and which are closer to reality. A hysteretic property of the generalized exergy describes a decrease of the maximum work received from an engine system and an increase of work added to a heat pump system, the features which are particularly important in high-rate regions of thermodynamic processes. For an infinite sequence of infinitesimal thermal machines, an optimal temperature strategy is obtained in the form similar to that known in the theory of simulated annealing.

1. Introduction: Thermoeconomic Aspect of Thermodynamic Optimization

In this paper we consider nonlinear dynamics of a nonideal engine system, Fig. 1, in which a hot fluid supplies pure heat to an engine at a high temperature $T = T_1$ and releases pure heat to an environmental fluid at a low temperature $T = T_2$. Such a process may be accomplished as a steady or unsteady one; we consider

*First presented at Meeting on Complex Systems in Natural and Economical Sciences, Hungary, Tata, 12 – 14 September, 1996.
the process at the steady state. For this process, which is, actually, an active (work-producing) heat exchange between two fluids, we consider the problem of maximum work delivered in a finite time. For an inverse process, in which the work is added and the system works as a heat pump, a related optimization problem is that of minimum work in a finite time.

We assume that both fluids conduct the heat according to Newton's law. Our modeling is restricted to the situation when the temperature of the second fluid is constant and equal to that of an environment. This means we assume an infinite bath of the second fluid, thus $T_2 = T_e$. We begin our analysis with the simplest case of a purely reversible (Carnot) system, without production of entropy. Next, two resistances are added. They link the heat sources with the working fluid of the engine at high and low $T$, as in the familiar Curzon-Ahlborn-Novikov engine [1–5]. Finally, a complex process arranged as a cascade composed of $N$ such differential CAN engines is analyzed. While a continuous model can be obtained from a corresponding discrete model, the inverse is not true, i.e., the discrete model (which is more general than continuous) does not follow from the continuous one, hence the substantiation for discrete modeling and optimization. We also show that the continuous limit of the cascade process constitutes a suitable theoretical tool to obtain a finite-time counterpart of the available energy (exergy) of the driving fluid. The classical availability is a well-known thermodynamic quantity which defines bounds on work from (and for) reversible processes [6]. Our result generalizes this classical idea for finite time transitions.

In the nonideal system in question, the efficiency of power production is smaller than the efficiency of the Carnot cycle operating between $T_1$ and $T_3$. However, the efficiency is merely one of the process characteristics, and by no means the quantity being maximized. In thermodynamics, it is rather total work which has to be maximal, whereas in thermoeconomics one usually maximizes a net profit. The latter is the difference between the profit brutto (resulting from work production) and the sum of the process operational and investment costs. A link between thermodynamic and economic optimization is discussed below.

2. General Problem of Work Optimization with a Constrained Investment

Any reasonable industrial enterprise requires the investment money to be used with some limitations, which means a constraint imposed on the investment. Whenever the investment is constrained, the system size and/or the residence time of flowing entities (e.g., heat sources, reacting species, etc.) must be finite, thus, for constrained investment costs, only a finite time is available to accomplish required changes of state in a real process. This property is however not taken into account in classical thermodynamics in which the reversible changes of states (associated with vanishing rates and infinite durations) imply an infinite investment. The property refers also to work-producing systems, especially thermal engines.