An Efficient Implementation of the Recursive Approach to Flexible Multibody Dynamics

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Abstract. This paper deals with the efficient extension of the recursive formalism (articulated body inertia) for flexible multibody systems. Present recursive formalisms for flexible multibody systems require the inversion of the mass matrix with the dimension equal to the number of flexible degrees of freedom of particular bodies. This is completely removed. The paper describes the derivation of equations of motion expressed in the local coordinate system attached to the body, then the discretization of these equations of motion based on component mode synthesis and FEM shape functions and, finally, two versions of the new recursive formalism.

Key words: flexible multibody system, recursive formalism, articulated body inertia, component mode synthesis, computational complexity.

1. Introduction

In the last decade great effort has been devoted to computer simulation of flexible multibody dynamics. The theoretical attitudes differ in applying the methods of the assembly of the system, the equations of motion and its solution. Generally, we can say that the methods separate the motion of a particle of a body into a motion of the body as a whole and into a small displacement due to deformation. This makes it possible to combine the methods developed for rigid body dynamics with methods used for solving the body deformation namely FEA. The disadvantage of the large number of deformation coordinates when applying FEA is removed by the “Component Mode Synthesis” introduced by Hurty [7]. However, this method involves an appropriate choice of a set of deformation modes which requires some experience. Usually the set of modes is a combination of so-called attachment modes, constraint modes and eigenmodes. Attachment modes are defined as modes solved from a static problem with a unit load acting on the degrees-of-freedom (DOF) of interest. Constraint modes are defined by prescribing unit displacements for some DOF. In [2] the equivalence of the attachment and constraint sets of modes has been proved.

Some methods use the Lagrange’s equations with multipliers in order to assemble the set of equations of motion and the obtained system of simultaneous
equations is solved, see [9–11]. Another attitude is a recursive method, which is the most efficient one for the rigid body systems from the point of view of computational complexity. The simplest formulation of this method is for an opened kinematic chain of bodies. The recursive method combines well the assembly of the equations of motion with their solution. The algorithm of this method requires the numerical integration to perform three stages in every time step. In the first one, called “first forward course”, the matrices of mass and force properties and the kinematic relations are computed for all bodies starting from the base. In the second stage, called “backward course”, starting from the last body of the chain, the second time derivatives of the body joint and modal coordinates are expressed and substituted into the rest of the equations. From the mathematical point of view it is a solution through substitution, from the physical point of view it is a reduction of the mass and force properties of the part of the chain from body \( i \) to the end, on the body preceding body \( i \). The third stage starts after the substitution method reduces the set of equations to a single equation, which is then solved. Then, in the so-called “second forward course”, starting from the first body to the last the accelerations are computed. This method was developed and applied for solving the direct dynamic problem by Kim and Haug [4]. In [3] they derived an equation of motion of a flexible body by applying the principle of virtual work. They transformed the integrals occurring to summations applying the “Lumped Mass Theorem”, which is commonly used by practically all other methods. In their algorithm, in each time step in the backward course, a matrix of order \((N + \sigma(i)), N + \sigma(i)\) must be inverted when eliminating the coordinates of body \( i \). Here \( N \) denotes the number of deformation modes and \( \sigma(i) \) denotes the number of DOF in the joint of body \( i \) with the preceding body \( p(i) \) in the chain. This leads to computational complexity proportional to the third power of the sum \((N + \sigma(i))\). One attempt at removing this disadvantage is given in [12], however, it is formulated in a global coordinate system and requires the multiplication of matrices of dimension \( N \). All these disadvantages have been removed in [13] by formulating the equations of motion in a local coordinate system attached to the body and by modifying the recursive formalism for flexible bodies.

In this paper three derivations from [13] are presented:
1. Derivation of equations of motion with all characteristics expressed in the local coordinate frame fixed to the body.
2. Discretization of the integrals in the equation of motion based on component mode synthesis and FEM shape functions.
3. A recursive algorithm for solving the equations of motion of a single kinematic chain of flexible bodies with reduced computational complexity.

2. Equation of Motion of a Flexible Body

We shall derive the equation of motion of a flexible body by applying the Gauss principle. We consider the body as a 3D continua and define a frame coordinate