Hard-core Scattering for $N$-body Systems

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Abstract. We prove propagation properties (maximal and minimal velocity bounds) for pseudo-resolvents associated to $N$-body Hamiltonians with short-range potentials that are infinite on a star-shaped domain centred at the origin. Motivated by the fact that the invariance principle holds for usual $N$-body systems, we define the cluster wave operators in terms of pseudo-resolvents and prove that they exist and are asymptotically complete. For any cluster decomposition $a$, these operators intertwine the hard-core pseudo-selfadjoint Hamiltonians corresponding to the pair of pseudo-resolvents $R, R_a$, and equal the Abel operators constructed in terms of Hamiltonians.

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1. Introduction

One of the most important goals in scattering theory is the study of the asymptotic behavior (when $t \to \pm \infty$) of $e^{-itH}\psi$, where $\psi$ is an arbitrary state from the orthogonal complement of the space of eigenvectors of the Hamiltonian $H$. More precisely, we are interested in finding a family $\{H_a\}$ of selfadjoint operators, with simpler (and known) spectral and evolution properties, such that, for any state $\psi$, a family of vectors $\{\psi_{a}^{\pm}\}$ should exist, for which the convergences

$$ \left\| e^{-itH}\psi - \sum_{a} e^{-itH_{a}}\psi_{a}^{\pm} \right\|_{t \to \pm \infty} \to 0 \quad (1.1) $$

are satisfied. If this takes place, then we say that the system is asymptotically complete.

The particularity of the $N$-body Hamiltonians is that they are a sum of a differential operator (with excellent dispersion properties) and a perturbation that does not vanish (when $|x| \to \infty$) along certain directions of the configuration space $X$. This makes us think that, if asymptotic completeness holds for such systems,

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then $e^{-itH_0} \psi_{\alpha}^\pm$ should be asymptotically localized within some cones centred on the classical trajectories.

These geometrical ideas allowed V. Enss to show that (1.1) is true for three-body quantum systems with potentials that decay slightly faster than the Coulomb interaction. After that, asymptotic completeness for $N$-body short-range quantum systems has been proved in 1987 by I. Sigal and A. Soffer [34], and in the following years, many people tried to simplify or to extend their proof for more complicated many-body problems. Indeed, there was first an effort of making the theory more ‘readable’, done by J. Dereziński in [14]. Then, G. M. Graf, jointly using ideas from the earlier works of Enss ([16, 17, 18, 19]) and from [34] but also from the more recent papers of Sigal and Soffer (like [36] and especially [35]), succeeded in giving in [23] a remarkable time-dependent-like proof of the quoted result, which differed from the previous proofs in several important aspects. We shall emphasize only the fact that in [23], some of the main propagation properties have been obtained without the use of the Mourre estimate, i.e., independently of an intimate knowledge of the spectral properties of the Hamiltonian. These properties were sufficient for showing the existence of the cluster wave operators but not for their completeness. Indeed, for the last result, a propagation property involving jointly a time-dependent localisation in position and a localisation in the total energy was needed, and for proving it a good knowledge of the spectrum of the $N$-body Hamiltonian was crucial. Actually, this is the only place where Graf invokes the Mourre estimate (in order to obtain (local) positivity for the commutator of the Hamiltonian with the generator of the dilations group) and by this means he eliminates the decay hypothesis imposed in [34] on the second derivative of the potential. Further, using refined results on the Mourre theory due to W. Amrein, Anne Boutet de Monvel and V. Georgescu (see [1] and also [11] for optimality) we have shown in [27], on the lines of [23], that no condition on the derivatives of the potential was needed in order to prove completeness for the Agmon-type systems. Moreover, since locally the potentials were allowed to be as singular as the the kinetic energy permits, the question of the validity of a statement on asymptotic completeness for much singularly perturbed systems (as the hard-core $N$-body quantum systems) arises naturally. Indeed, the interest for such problems is rather old, going back, e.g., to the works of W. Hunziker ([26]) and especially of D. W. Robinson, P. Ferrero and O. de Pazzis (see [32, 21]), where, under rather restrictive assumptions on the geometry of the potentials (spheric symmetry, the supports of the singularities where cylinders centred on the subspaces of the relative movement of the clusters) and on the forces (repulsivity), the absence of the singular continuous spectrum and the existence and the completeness of the wave operators corresponding to the elastic channel have been established. But this is, of course, a very simplified case, because even if the problem was posed in an $N$-body context, the above hypotheses transformed it in a one-channel scattering problem. Very recently, Anne Boutet de Monvel, V. Georgescu and A. Soffer, using both the locally conjugate operator method and an algebraic approach (which appears naturally in the $N$-