Deformation of modulated wave groups in third-order nonlinear media*

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Abstract. We present a numerical and analytical investigation of the deformation of a modulated wave group in third-order nonlinear media. Numerical results show that an optical pulse that is initially bi-chromatic can deform substantially with large variations in amplitude and phase. For specific cases, the bi-chromatic pulse deforms into a train of temporal solitons. Based on the coupled phase-amplitude equation of Nonlinear Schrödinger (NLS), the initial deformation of the modulated wave-packet will be explained and an instability condition can be derived. Energy arguments are given that provide an alternative derivation of the instability condition.

Key words: nonlinear Schrödinger equation, phase-amplitude equation, dispersion trajectory, nonlinear dispersion relation

1. Introduction

As is well known, nonlinear effects can present itself in large deformations of pulses or beams. The deformation of an initially monochromatic wave due to modulational instability (Benjamin–Feir instability) is characteristic; the deformation into a soliton profile is well described (Akhmediev 1997) and exploited to produce solitons in practical situations (Hasegawa 1984).

In this paper we investigate the deformation of bi-chromatic waves and show in various ways that a recurrent deformation into first or higher order soliton-shapes, is an inherent consequence when nonlinearity is strong enough but balanced by dispersion as described by the characteristic Nonlinear Schrödinger (NLS) equation. Different from the Benjamin–Feir instability of a monochromatic wave, the envelope of the bi-chromatic wave vanishes at the interference nodes, which leads to large variations in the envelope and properties of the underlying carrier wave. Similar properties appear during interaction of bi-soliton solutions; the study of this non-

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periodic case is simplified by the fact that exact expressions are available, see van Groesen et al. (this issue). In the periodic case to be considered here, we present numerical calculations and various analytical arguments to arrive at comparable results. In both cases, the characteristic parameter is the quotient of amplitude and the frequency difference (which is inversely proportional to the modulation period). For the bi-soliton interaction, the interaction length is inversely proportional to the amplitude of the soliton, while for a periodic wave train the interaction region is the total periodic interval.

The organization of the paper is as follows. First, in Section 2, the governing NLS equation is derived for pulses and beams in material with third-order nonlinearity; the detailed derivation specifies the transformation from physical to normalized variables, which makes it possible to identify the physical relevance of the phenomena to be derived. Besides the standard equation for the complex amplitude, also the phase–amplitude equations are given for later explanation of the basic phenomena. In Section 3 we present the results of numerical calculations and show the appearance of large deformations in envelope and carrier wave. We show that the envelope can deform to a good approximation in a periodic train of solitary waves. In Section 4 we present three independent arguments to explain the observed deformations when a characteristic parameter exceeds a lower bound. This parameter, i.e. the quotient of wave amplitude and frequency difference of the carrier waves, indicates that smaller frequency differences have the same effect as larger amplitudes. Conclusions and discussions complete the paper.

2. Governing equation

2.1. TEMPORAL-NLS EQUATION FOR PULSE PROPAGATION

We consider the propagation of optical pulses in a single mode fiber and write the effective electric field in the form

\[ \vec{E}(z,t) = \frac{1}{2} \{ A(z,t) \exp(i(k_0 z - \omega_0 t) + h.o.t + c.c.) \} \cdot \vec{x}, \]

where \( \vec{x} \) is the polarization unit vector, \( k_0 = K(\omega_0) = (\omega_0/c)n_0 \) is the propagation constant (wave number) at the carrier wave frequency \( \omega_0 \) and \( A(z,t) \) is the slowly varying complex envelope of the effective electric field. The refractive index of the fiber medium is taken to be

\[ n(\omega) = n_0(\omega) + n_2(\omega)|E|^2, \]