Upscaling Uncertain Permeability Using Small Cell Renormalization

J. J. Hastings\textsuperscript{2} and A. H. Muggeridge\textsuperscript{2}

Sedimentary rocks have structures on all length scales from the millimeter to the kilometer. These structures are generally associated with variations in rock permeability. These need to be modeled if we are to make predictions about fluid flow through the rock. However, existing computers are not powerful enough for us to be able to represent all scales of heterogeneity explicitly in our fluid flow models—hence, we need to upscale. Small cell renormalization is a fast method for upscaling permeability, derived from an analogue circuit of resistors. However, it assumes that the small scale permeability distribution is known. In practice, this is unlikely. The only information available about small scale properties is either qualitative, derived from the depositional setting of the reservoir, or local to the wells as a result of coring or logging. The influence of small scale uncertainty on large scale properties is usually modelled by the Monte Carlo method. This is time-consuming and inaccurate if not enough realisations are used. This paper describes a new implementation of renormalization, which enables the direct upscaling of uncertain small-scale permeabilities to produce the statistical properties of the equivalent coarse grid. This is achieved by using a perturbation expansion of the resistor-derived equation. The method is verified by comparison with numerical simulations using the Monte Carlo method. The prediction of expected large-scale permeability and its standard deviation are shown to be accurate for small cell standard deviations of up to 40\% of the mean cell value, using just the first nonzero term of the perturbation expansion. Inclusion of higher order terms allows larger standard deviations to be modeled accurately. Evaluation of cross-terms allows correlations of actual cell values, over and above the background structure of mean cell values. The perturbation method is significantly faster than conventional Monte Carlo simulation. It needs just two calculations whereas the Monte Carlo method needs many thousands of realisations to be generated and renormalized to converge. This results in significant savings in computer time.

KEY WORDS: Monte Carlo, perturbation, aquifer, heterogeneity, stochastic.

INTRODUCTION

Hydrocarbon reservoirs and aquifers are found in sedimentary rocks. These rocks have structures on all length scales from the millimeter (e.g., crossbedding) to the kilometer (e.g., channels). Rock permeability also varies on all length scales.

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\textsuperscript{2} T. H. Huxley School of Environment, Earth Sciences & Engineering, Royal School of Mines, Imperial College, London SW7 2BP. e-mail: j.hastings@ic.ac.uk
This heterogeneity may be associated with visible depositional structures or less visible, more random variations due to the sorting of the grains or post depositional diagenesis.

To predict the fluid flow in hydrocarbon reservoirs and aquifers, we need to be able to describe the permeability distribution within them. Unfortunately, this presents us with two problems:

1. we don’t know the exact permeability distribution in the subsurface on all length scales, and
2. even if we did, we can’t model it explicitly in our numerical simulations because of insufficient computer speed and memory.

In the oil industry these problems are normally addressed by a combination of upscaling and Monte Carlo simulation. Numerous, equiprobable geological models of the reservoir are created. These are then populated with porosity and permeability. Each model is then upscaled using a deterministic method (see, for example, the review of Renard and de Marsily, 1997). Fluid flow simulation is then used to determine the recovery, etc., associated with each model. The results are compiled to give probability distributions of recovery.

This is inevitably very time-consuming. Accuracy of the probability distributions is also questionable, as many engineers simply do not have enough time to generate sufficient realizations to make the Monte Carlo method valid. In addition, it is very difficult to determine the impact of different aspects of the permeability distribution on recovery when applying the Monte Carlo method.

Recently attention has been focused on the development of stochastic flow simulation (for example, Dagan, 1986; Dainton, Nichols, and Goldwater, 1997; Zhang, 1998). This incorporates the effect of uncertainty directly in the fluid flow simulation, avoiding the need to generate and simulate flow in multiple realizations of the reservoir. However, upscaling becomes more important as these models are even more intensive in their use of computer resources than deterministic simulators. In addition, they need both the expected value of permeability and its variation for each grid block at their coarse simulation scale. Clearly this could be obtained via deterministic upscaling and the application of the inevitable Monte Carlo simulation. However, the problems associated with Monte Carlo methods, described above, would still apply.

This paper considers the problem of direct single phase upscaling of stochastic reservoir models, i.e., methods that do not require the generation of multiple reservoir realizations to describe the uncertainties about the reservoir. These methods can be used to generate input data for stochastic reservoir simulation or simply to evaluate the impact of small-scale uncertainty on large-scale permeabilities, prior to deterministic reservoir simulation.

The method uses a perturbation expansion of the equation for effective permeability obtained by the “Small Cell Renormalisation” (SCR) approach (King,