Plane crack propagation in a hyperelastic incompressible material

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Abstract. We propose a crack propagation criterion for hyperelastic materials (rubber type material) within the framework of plane elasticity in finite deformation. The criterion is based on the examination of the asymptotic elastic field near the crack tip prior to propagation. According to this criterion, the propagation will take place for a critical value of the strain energy density intensity factor. The kink angle, obtained by applying the criterion of maximum opening stress, will depend on the fracture tensile stress of the actual material. We propose to use a local iterative finite element method to compute the asymptotic quantities involved in the criterion at a reasonable cost. Examples of computation for some hyperelastic laws simulating the behavior of vulcanized rubber are presented.

Key words: Crack, criterion of propagation, hyperelasticity, iterative finite element method, opening stress, rubber, strain energy density.

1. Introduction

In the classical context of brittle fracture mechanics (concerning materials like glass or some brittle metals for example), the crack propagation in a body takes place without notable unrecoverable deformation near the crack tip (confined plasticity). After relaxation of the loading, the body recovers almost its initial shape but with a larger crack. The fracture is then only governed by the near tip elastic fields under small strain and small rotation hypothesis (linear or classical elasticity).

Figure 1. Plane cracked domain.
In this paper we are concerned with the problem of fracture of rubber type or elastomeric materials. We can observe that these materials have a similar fracture behavior except that they are able to undergo significant recoverable deformation before failure so that the hypothesis of small perturbations will be relinquished. Hyperelastic constitutive laws can correctly model the behavior of these materials.

We propose here a criterion of propagation which is an adaptation of the basic concepts of brittle fracture mechanics for finite deformation elasticity.

Let us recall briefly the main classical theories available and the statement of the corresponding propagation criteria. We consider a body occupying a cylindrical domain (of thickness unity) initially cracked, in equilibrium under the effect of a given loading supposed to cause the opening of the crack. The framework is that of plane elasticity for a homogeneous and isotropic body in its reference (free of stress) or undeformed configuration. A material point of a plane section of the undeformed body, normal to the generators of the cylinder, is located by its polar coordinates \((r, \theta)\), \(0 \leq \theta \leq 2\pi\) relative to the reference frame \(OX_1X_2\) where \(O\) is the tip of the rectilinear crack of direction \(OX_1\) (see Figure 1).

The classical criteria state that the propagation will take place as soon as some intensity factor associated with the asymptotic elastic fields developed near the crack tip reaches an experimental critical value. It is known that the stress field is singular in \(r = 2\) and depends on two intensity factors \(K_I\) and \(K_{II}\) (its well-known expression will not be pointed out here). The opening mode or mode I corresponds to \(K_{II} = 0\), the slip mode or mode II to \(K_I = 0\) and the mixed mode for the other cases.

We will distinguish two types of criteria, the first one requires only the knowledge of the current asymptotic fields (before propagation) whereas the second one takes into account the fields right after the onset of propagation. In the first category we find the criteria of Griffith, Irwin, Erdogan and Sih, and Sih. For the two first, only applicable in mode I for which there is no crack branch, the propagation takes place as soon as, respectively, the energy release rate \(G\) and the factor \(K_I\) reach their critical value \((G = -\lim_{\Delta\ell \to 0} \Delta E_P/\Delta\ell\), where \(E_P\) is the