A hospital facility layout problem finally solved

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This paper presents a history of a difficult facility layout problem that falls into the category of the Koopmans–Beckmann variant of the quadratic assignment problem (QAP), wherein 30 facilities are to be assigned to 30 locations. The problem arose in 1972 as part of the design of a German University Hospital, Klinikum Regensburg. This problem, known as the Krarup 30a upon its inclusion in the QAP library of QAP instances, has remained an important example of one of the most difficult to solve. In 1999, two approaches provided multiple optimum solutions. The first was Thomas Stützle’s analysis of fitness–distance correlation that resulted in the discovery of 256 global optima. The second was a new branch-and-bound enumeration that confirmed 133 of the 256 global optima found and proved that Stützle’s 256 solutions were indeed optimum solutions. We report here on the steps taken to provide in-time heuristic solutions and the methods used to finally prove the optimum.

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1. The problem

Spadille Inc., Consultants of Industrial Statistics and Operations Research was established in 1970. Jakob Krarup was one of the three co-founders and served as a manager of this still existing Danish company until his return to academia in 1975.

Among the projects undertaken was the design of a university hospital, Klinikum Regensburg, to be built in Regensburg, Germany. In 1972, an invitation to submit tenders was issued to a number of architects. As was explicitly stated in the announcement of the competition, among the criteria to be considered in the evaluation of proposals was the usual QAP objective: find a layout minimizing the sum CDIST of Communication $\times$ DISTance taken over all pairs of facilities to be located.

To assess the architects’ proposals regarding this criterion only, Spadille’s task was to find “a best lower bound” on CDIST. Or, if at all possible, Spadille was to find provably optimal solutions to a series of instances of the Koopmans–Beckmann variant of the more general QAP. Included were two instances which later were to be placed in a quadratic assignment problem library (QAPLIB) established in 1991 by Burkard et al. (1991) as the “Krarup 30a/b”.

Let $n$ be the number of facilities to be located in $n$ out of $m$ cells, $m > n$. For a hospital, facilities can be viewed as equally sized objects or “functions” such as surgery, X-ray, etc. For our purpose, we can consider a cell as any point with positive integer coordinates in a 3-dimensional co-ordinate system. Each cell can accommodate exactly one of the $n$ facilities to be located.

It can be argued that the two cases “$m = n$” and “$m > n$” essentially are equivalent since $m - n$ “dummy” facilities can be added if $m$ is strictly greater than $n$. From a practitioners’ point of view,
however, these two cases may well represent different situations.

With the straightforward interpretation of QAP in mind (assignment of facilities to cells within a building), \( m = n \) means that the building is designed to exactly house the \( n \) facilities in its free cells. Of course, there may also be other cells in the building, but they are in this context regarded as inadmissible, perhaps because they are reserved for other purposes.

The ‘‘\( m > n \)’’-case is relevant when none or only some information is available as to the exact shape of the building. For example, for Klinikum Regensburg, the only restrictions imposed were due to the surrounding landscape. Thus, a beautiful pond in the park had to be protected. Other aesthetic criteria to be taken into account were reflected by an upper bound (two) on the number of floors. The shape of the building was not otherwise determined a priori, but was supposed to result from the ‘‘most compact arrangement of the \( n \) facilities’’ suggested by the architects or determined by solving the corresponding QAP.

2. Spadille’s approach to solving the problem

A number of instances with \( n \) ranging from 30 to 48 were investigated. Since no algorithm at that time (1972) was capable of solving such sizeable instances to optimality, Spadille could hope for nothing better than generating lower bounds, hopefully below the objective function values reached by the architects.

To this end, a randomized heuristic was devised (Krarup, 1972; Krarup and Pruzan, 1978). Let \( x_u, y_u, z_u \) be upper bounds on the length, the width, and the number of floors and let \( m = x_u y_u z_u \). Initially, the \( n \) facilities were placed at random in \( n \) out of the \( m \) cells where the bounds \( x_u, y_u, z_u \) were chosen such that \( m \gg n \). One department at a time, say department \( D_i \), was then lifted from its present position and placed in a cell \( C_k \) representing the optimal solution to the ‘‘weighted 1-median’’. The weighted 1-median problem was defined by the positions of the remaining \( n - 1 \) facilities and with weights representing their communication with \( D_i \). In the case cell \( C_k \) already was taken up by another department, then the nearest cell among the \( m - n + 1 \) free cells was chosen. This step was repeated for all \( n \) facilities taken in a random order. Afterwards, an interchange procedure was followed until a kind of ‘‘2-optimality’’ was achieved.

That is, for the current layout of the \( n \) facilities, no further reduction of the objective function value CDIST would result from interchanging any pair of facilities.

Each run as described above resulted in a layout and a corresponding upper bound on the (unknown) minimum value CDIST\(_{\text{min}}\) of CDIST. The series of such runs was continued until the relative difference between the best and the second best solution found was below a pre-specified threshold value or a certain time limit was exceeded.

For each instance of QAP examined, a set of the 10 best layouts found were presented to the contractor together with the corresponding values of CDIST. Based on these 10 values of CDIST, Spadille was furthermore requested to produce a good estimate of CDIST\(_{\text{min}}\). Theoretical studies of QAP conducted much later would have confirmed Spadille’s suspicion of a very flat optimum in general. Disregarding that suspicion, however, Spadille had no idea as to the real ‘‘landscape of optimal and near-optimal solutions’’.

In hindsight, Krarup today is in no way proud of the statistical tools used to estimate CDIST\(_{\text{min}}\). However, the contractor was satisfied and very pleased to see that no architect managed to come up with a layout beating Spadille’s layout in terms of CDIST.

In addition to \( n \) and \( x_u, y_u, z_u \), the input data for each run consisted of a symmetric \( n \times n \) matrix \( C = (c_{ij}) \) specifying the communications between each pair of facilities. Finally, for a pair \((D_i, D_j)\) of cells with coordinates \((x_i, y_i, z_i)\) and \((x_j, y_j, z_j)\) respectively, the distance \( d_{ij} \) between these was calculated as

\[
d_{ij} = c_1 \times |x_i - x_j| + |y_i - y_j| + c_2 \times |z_i - z_j|
\]

where \( c_1 \) and \( c_2 \) are constants reflecting average transport + waiting time incurred by horizontal and vertical movements in the building. The great computational advantage of the Manhattan distance measure employed is that each execution of weighted 1-median, the core of the randomized heuristic, requires \( O(n) \) steps only.

3. Early computational results

For the Klinikum Regensburg problem with \( n = 30 \), Spadille used \( c_1 = 50, c_2 = 115 \) in the distance function above. Krarup has no recollection of the exact values of \( x_u, y_u, z_u \). However, they were all ‘‘sufficiently large”, perhaps as large as 10. The best