PROBLEM ON FORMATION OF PARALLEL CRACK SYSTEM IN BRITTLE LAYER

A. Ph. Revuzhenko and S. V. Klishin

The process is considered for crushing a brittle layer. The finite-element method and a number of simplified models associated with the averaging are used for calculating the strains and stresses. The chief aim of investigation is to select such value for the parameter of loading under which the failure condition is achieved inside the layer. It is shown that its value depends both on the ratio of layer linear-dimension and the values of material elastic characteristics.

Brittle layer, deformation, failure, fracturing, finite-element method, energy flows

Currently, it is established that rocks are the block medium intersected by cracks of different scale levels; in addition to it, the position, orientation, and other parameters of every single crack are stochastic. However, the distinct and determinate regularities have been traced back to all the averaged characteristics of cracks. Investigation into fracturing of the rock mass is an actual problem for mining, construction of underground structures, hydraulic engineering, etc. In this connection, an intensive search is made for the solutions of different problems on fracturing by means of geological methods and methods of deformable solid mechanics [1–3]. In [4], the imitative model is considered for cracking a plane layer subjected to the biaxial nonuniform tension. This model makes it possible to obtain numerically different polygonal structures according to the prescribed parameters. The question concerning the role of actual physical parameters of the rock mass requires additional study.

Let us examine the deformation of thin layer. Assume that its material is ideally brittle and is deformed elastically prior to failure. As a failure criterion, we take the factor when the highest tensile stress reaches the given value. We analyze the problem in two statements: in the first simplified problem, the load is applied to both surfaces (upper and lower) of the layer, and in the second — only to the lower one. The problems are three-dimensional. However, in order to simplify all calculations, we can use the fact that the layer thickness is much less than its cross-dimensions. First, let us consider the problem of uniaxial tension in two-dimensional statement. Then, we reduce it to one-dimensional by means of averaging with respect to layer thickness and make sure that the solution error is small. Let us next carry the results of averaging over to more general three-dimensional case.

Consider the Cartesian coordinate system \((x, z)\) and homogeneous isotropic elastic rectangular domains: \(-l \leq x \leq l, -h \leq z \leq h\) (Fig. 1a) and \(-l \leq x \leq l, 0 \leq z \leq 2h\) (Fig. 1b), where \(L = 2l\) is the layer length, and \(H = 2h\) is its width.

---

Assume that the volume forces are absent, the vertical boundaries are free from the stresses, and the prescribed displacements are applied to the bases. We consider two problems with boundary conditions:

\[
\begin{align*}
    z = -h \text{ and } z = h \quad & u(x, z) = k_1 x, \ w(x, z) = 0 \\
    z = 0 \quad & u(x, z) = k_2 x, \ w(x, z) = 0, \\
    z = 2h \quad & \sigma_{xz}(x, z) = \sigma_{zz}(x, z) = 0,
\end{align*}
\]

(1)

(2)

Here, \( u(x, z) \) and \( w(x, z) \) are the displacement vector components; \( \sigma_{xx}(x, z), \sigma_{xz}(x, z), \) and \( \sigma_{zz}(x, z) \) are the stress tensor components; \( k_1 \) and \( k_2 \) are the time functions under quasi-static loading.

Let us introduce dimensionless values: \( x = H \bar{x}, \ z = H \bar{z}, \ u = H \bar{u}, \) and \( w = H \bar{w}, \) where \( H \) is the length scale, and the corresponding dimensionless variables are denoted by the over-bar. In order to choose the scale of stresses, we examine the problem of simple tension when the normal tensile stresses \( \sigma_{xx} = \sigma^* = \text{const} \) are assigned on \( AB \) and \( CD \) (Fig. 1c), and the bases \( AD \) and \( BC \) are free from stresses. This problem — \( \sigma_{xx} = \sigma^*, \ \sigma_{xz} = \sigma_{zz} = 0 \) — is solved over the whole domain. Let the layer undergoes a rupture on the \( z \) axis at a certain prescribed value of \( \sigma^* \). We take the value of \( \sigma^* \) as the scale of stresses: \( \sigma_{xx} = \sigma^* \bar{\sigma}_{xx}, \ \sigma_{zz} = \sigma^* \bar{\sigma}_{zz}, \) and \( \sigma_{xz} = \sigma^* \bar{\sigma}_{xz}. \) Hereafter, we shall omit the bar over the variables.

With new variables, the equilibrium equations and Hook’s law will have the following form:

\[
\begin{align*}
    \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{xz}}{\partial z} &= 0, \\
    \frac{\partial \sigma_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} &= 0, \\
    \sigma_{xx} &= \frac{\lambda \theta}{\sigma^*} + \frac{2\mu}{\sigma^*} \frac{\partial u}{\partial x}, \\
    \sigma_{zz} &= \frac{\lambda \theta}{\sigma^*} + \frac{2\mu}{\sigma^*} \frac{\partial w}{\partial z}, \\
    \sigma_{xz} &= \frac{\mu}{\sigma^*} \left( \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} \right),
\end{align*}
\]

where \( \theta = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}; \ \lambda \) and \( \mu \) are the Lamé parameters.

**Solution by the Finite-Element Method**

For problem (1), the layer after deformation is shown in Fig. 2a, and for (2), it is illustrated in Fig. 2b at the ratio \( L / H = 2 \). Triangular elements, a part of which is demonstrated in the figures, were chosen as a grid for the finite-element method. The number of the grid nodes was taken to be proportional to the value of \( L / H \).