Differential cross-sections of muonic atoms scattering

Asymmetric collisions: \( p\mu + (d, t), \quad d\mu + (p, t) \) and \( t\mu + (p, d) \)

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We present the differential cross-sections \( d\sigma(E, \Omega)/d\Omega \) for elastic and inelastic (isotopic exchange) scattering of muonic hydrogen, deuterium and tritium on hydrogen isotopes nuclei for the case of difference in masses of the projectile \( \mu \)-atom and the target nucleus. Available partial phase shifts have been used in the calculations and the results are presented in tables and figures for different CMS collision energies. The cross-sections are important for description of the slowing down and diffusion of \( \mu \)-atoms in matter and particularly for description of kinetics of muon catalyzed nuclear fusion.

1. Introduction

In our previous paper [1] the computed differential cross-sections for elastic and inelastic scattering of hydrogen, deuterium and tritium \( \mu \)-atoms in the ground states on \( p, d, \) and \( t \) nuclei, respectively, have been reported for symmetric cases of collisions:

\[
\begin{align*}
\mu(F_i) + p &\rightarrow \mu(F_j) + p, \\
\mu(F_i) + d &\rightarrow \mu(F_j) + d, \\
\mu(F_i) + t &\rightarrow \mu(F_j) + t,
\end{align*}
\]

for a full set of total orbital angular momenta, total spins of a three-particle system (\( \mu \)-atom + a nucleus) and for all possible initial, \( F_i \), and final, \( F_j \), spins of \( \mu \)-atoms.

The knowledge of differential cross-sections is necessary for describing a broad spectrum of different phenomena in muon physics [2,3]. Among these the slowing down, thermalisation and diffusion of \( \mu \)-atoms in mixtures of hydrogen isotopes are of great interest from the point of view of the muon catalyzed fusion kinetics studies [4–6].

In order to describe the transport of \( \mu \)-atoms in targets containing mixtures of different hydrogen isotopes it is also necessary to know the differential scattering

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cross-sections for asymmetric cases:

\[
\begin{align*}
\tau \mu + d &\rightarrow \tau \mu + d, \quad \delta \mu + t \rightarrow \delta \mu + t, \\
\delta \mu + p &\rightarrow \delta \mu + p, \quad \rho \mu + d \rightarrow \rho \mu + d, \\
\tau \mu + p &\rightarrow \tau \mu + p, \quad \rho \mu + t \rightarrow \rho \mu + t
\end{align*}
\]

(2)

(elastic scattering) and

\[
\begin{align*}
\tau \mu + d &\rightarrow \delta \mu + t, \quad \rho \mu + d \rightarrow \delta \mu + p, \\
\delta \mu + t &\rightarrow \tau \mu + d, \quad \tau \mu + p \rightarrow \rho \mu + t, \\
\delta \mu + p &\rightarrow \rho \mu + d, \quad \rho \mu + t \rightarrow \tau \mu + p
\end{align*}
\]

(3)

(isotope exchange processes). Presentation of such cross-sections is the aim of this paper.

Recently, the values of the reactance matrix \( t_{ij}^J(E) \) and corresponding partial cross-sections \( \sigma_{ij}^J(E) \) for reactions (2), (3) have been published [7] for different values of the total orbital momentum \( J \). The results [7] have been obtained in the adiabatic representation [8], where the Coulomb three-body problem is reduced to the multichannel scattering with one or two open channels and a great number (~250) of closed ones [9,10]. So far these calculations are estimated as more accurate.

Using the available values of phase shifts and an algorithm of computation similar to that reported in [1] we calculated the differential cross-sections for processes (2), (3) in a wide range of CMS collision energies (up to a few hundreds of eV). It is worth mentioning that the calculations are done for the scattering of \( \mu \)-atoms on bare nuclei, so for energies below \( \approx 1 \) eV the necessary corrections for the electron screening and for molecular effects would have to be introduced [11]. In contrary, an evaluation of these effects [12] shows that they are negligible, above the epithermal region \((E > 1 \text{ eV})\), in comparison with the errors appearing due to uncertainties in the initial values of \( t_{ij}^J(E) \). In such cases presented differential cross-sections may be used without the above mentioned restrictions.

2. Formulae

When considering reactions (2) and (3), where the mass \( M_a \) of the muonic atom \( a \mu \) is different from the mass \( M_b \) of the b nucleus \((a, b \equiv p, d, t)\) the system of three particles, \( a-\mu-b \), for large internuclear distances, can be considered as a pair of states (figure 1).

We label with index ‘1’ the lower state, \( a \mu + b \), with the mass \( M_a \) of the muonic atom \( a \mu \) greater than the mass \( M_b \) of the nucleus \( b \). The higher state ‘2’ \((b \mu + a)\) is separated by the energy gap \( E_{\text{thr}} \), which is equal to the isotopic shift of the ground state energy level of a muonic atom due to the replacement of nucleus \( a \) with nucleus \( b \). The energies \( E_1 \) and \( E_2 \) are CMS collision energies in lower and upper channels, respectively. In such a system the elastic scattering \( 1 \rightarrow 1, 2 \rightarrow 2 \) and isotopic exchange \( 1 \rightarrow 2, 2 \rightarrow 1 \) are possible, depending on the collision energy (the transition