DETERMINATE SYSTEMS

The 2F3D-Model of Filtration in the Bottom Zones of Horizontal Oil Wells. II
A. V. Akhmetzyanov and V. N. Kulibanov

Trapeznikov Institute of Control Sciences, Russian Academy of Sciences, Moscow, Russia

Received May 15, 2001

Abstract—The difference approximation in time and space of the original continuous equations (Part I) was discussed. The existence conditions for the corresponding algebraic equations were presented. The solution of the difference equations was proved to converge to that of the original equations of the continuous model. The problem of identification of the basic parameters of the finite-difference model was discussed.

1. INTRODUCTION

For the problem of distribution in time and space of the functions of pressure and oil saturation upon filtration of oil and water in porous medium, the qualitative results on existence and smoothness of its solution (Part I) can be used to develop efficient numerical methods. The present paper considers the popular numerical method of alternating fixation at the time steps of the functions of pressure and oil saturation. At each step of the procedure, one has to solve a system of nonlinear algebraic equations. The paper proves that under certain conditions there exists a bounded solution of these equations. The solution of the finite-difference equations is proved to converge to that of the original continuous system. The proposed mathematical model involves some parameters and functions whose forms and values cannot be fully established from the literature and other external sources. Therefore, to make the mathematical models adequate to the physical processes, it is advisable to use procedures for their identification by the redundant information from the history of oil field development.

2. DIFFERENCE SCHEMES TO SOLVE THE ORIGINAL EQUATIONS

Let us consider the following approximate method of solving problem “A.” We decompose the time interval \([t_0, T]\) into \(N\) layers of width \(\tau = (T - t_0)/N\) and solve in each \((n + 1)\)th layer the following equation system (under the given boundary conditions (6), (7) from [1]):

\[
\frac{\partial P^{n+1}(x,t)}{\partial t} = \text{div} \left[ K_0(\sigma^n) \text{grad} P^{n+1} \right], \tag{1}
\]

\[
\frac{\partial \sigma^{n+1}(x,t)}{\partial t} = \frac{\varepsilon}{m} \text{div} \left[ m \mu_0 \left( \frac{1}{\mu_0} \frac{\partial K_0(\sigma^n)}{\partial \sigma} - \frac{c_0 \sigma^n K_0(\sigma^n)}{m \sigma^n + c_0 + c_w (1 - \sigma^n)} \right) \right] \times \text{grad} P^{n+1} + \left[ \frac{1}{m \mu_0} \frac{\partial K_0(\sigma^n)}{\partial \sigma} - \frac{c_0 \sigma^n}{m \sigma^n + c_0 + c_w (1 - \sigma^n)} \right] \left( \text{grad} P^{n+1} \right), \tag{2}
\]

\[
\frac{d \eta^{n+1}}{dx_1} = \frac{\lambda_{\text{mix}} W_{\text{mix}}^{n+1}}{2d} \left| W_{\text{mix}}^{n+1} \right| \left( \rho_0 \varphi_0^{n+1} + \rho_w \varphi_w^{n+1} \right). \tag{3}
\]
The following notation is used in (1)–(3):

\[ n \tau \leq t \leq (n + 1) \tau, \quad W_{n+1}^n = \left( Q_0^{n+1} + Q_w^{n+1} \right)/f, \]

\[ Q_0^{n+1} = \int_{S(x_1)} \frac{K_0(\sigma^n)}{\mu_0} \left( \mathbf{K} \text{ grad } P^{n+1} + \hat{\mathbf{v}} \right) ds, \]

\[ Q_w^{n+1} = \int_{S(x_1)} \frac{K_w(\sigma^n)}{\mu_w} \left( \mathbf{K} \text{ grad } P^{n+1} + \hat{\mathbf{v}} \right) ds, \]

\[ \varphi_0^{n+1} = Q_0^{n+1}/ \left( Q_0^{n+1} + Q_w^{n+1} \right), \quad \varphi_w^{n+1} = Q_w^{n+1}/ \left( Q_0^{n+1} + Q_w^{n+1} \right), \]

\[ \sigma^n(x, n \tau) \equiv \sigma^n(x), \quad P^n(x, n \tau) \equiv P^n(x). \]

The following initial conditions are valid in each layer:

\[ \sigma^{n+1}(x, n \tau) = \sigma^n(x), \quad P^{n+1}(x, n \tau) = P^n(x), \]

\[ \sigma^{(0)}(x) = \sigma_{\text{beg}}(x), \quad P^{(0)}(x) = P_{\text{beg}}(x). \]

At the \((n + 1)\)th step of the iterative procedure, solutions of the function \(\sigma^n(x), P^n(x)\) are already regarded as known. Let us consider a possible difference scheme realizing the proposed method.

\[ \left( P_{n+1}^{ijk} - P_{n}^{ijk} \right) \frac{h^2}{\tau} = \frac{1}{m_{ijk}[c_o \sigma_{n}^{ijk} + c_w(1 - \sigma_{n}^{ijk})]} \]

\[ \times \left[ K_0(\sigma_{n}^{ijk})K_{11ij} \left( P_{n+1}^{ij+1k} - P_{n}^{ij+1k} \right) \frac{h^2}{\Delta y_{i+1j} \Delta \bar{y}_{ij}} - K_0(\sigma_{n}^{i-1j})K_{11i-1j} \left( P_{n+1}^{ij-1k} - P_{n}^{ij-1k} \right) \frac{h^2}{\Delta x_{i-1j} \Delta \bar{x}_{ij}} \right] \]

\[ \times \left( P_{n+1}^{ij+1k} - P_{n}^{ij+1k} \right) \frac{h^2}{\Delta y_{ij+1k} \Delta \bar{y}_{ij}} - K_0(\sigma_{n}^{ij-1})K_{33ij-1} \left( P_{n+1}^{ij-1k} - P_{n}^{ij-1k} \right) \frac{h^2}{\Delta z_{ij-1k} \Delta \bar{z}_{ij}} \]

\[ \times \frac{h^2}{\Delta y_{ij} \Delta \bar{y}_{ij}} + K_0(\sigma_{n}^{ij})K_{33ij} \left( P_{n+1}^{ij+1k} - P_{n}^{ij+1k} \right) \frac{h^2}{\Delta z_{ij+1k} \Delta \bar{z}_{ij}} \]

\[ -K_0(\sigma_{n}^{ij-1})K_{33ij-1} \left( P_{n+1}^{ij-1k} - P_{n}^{ij-1k} \right) \frac{h^2}{\Delta z_{ij-1k} \Delta \bar{z}_{ij}} \].

(4)

Here, \(K_{11ij}, K_{22ij}, \text{ and } K_{33ij}\) are the values of the diagonal elements of the matrix \(K\) at the nodes of the difference grid \(ijk\); \(\Delta x_{ijk}\) is the step of the difference grid between the nodes \(ijk\) and \((i - 1)jk\); \(\Delta \bar{x}_{ij} = (\Delta x_{i+1j} + \Delta x_{ij})/2, \sum \Delta x_{ijk} = L_x\) is the length of the parallelepiped edge along the axis \(x\) \((\Delta y_{ij}, \Delta \bar{y}_{ij}, \Delta z_{ij}, \Delta \bar{z}_{ij})\) are defined in a similar manner; \(h\) is the characteristic size of the steps of the difference grid; the initial and subsequent time distributions of pressure at the nodes of the difference grid \(ijk\) follow the relationships

\[ P_{n+1}^{ijk} = P_{n+1}^{\text{bed}ijk}, \quad ijk \in S_{p\Delta}, \]

\[ P_{n}^{ijk} = P_{n+1}^{\text{bed}ijk}, \quad ijk \in S_{p\Delta}, \]

\[ P_{n+1}^{ijk} = P_{n+1}^{\text{bed}ijk}, \quad ijk \in S_{p\Delta}, \]

\[ P_{n}^{ijk} = P_{n+1}^{\text{bed}ijk}, \quad ijk \in S_{p\Delta}, \]

\[ P_{n+1}^{ijk} = P_{n+1}^{\text{bed}ijk}, \quad ijk \in S_{p\Delta}, \]

\[ P_{n}^{ijk} = P_{n+1}^{\text{bed}ijk}, \quad ijk \in S_{p\Delta}, \]

(5)

where \(S_{p\Delta}\) is the set of the nodes on the parallelepiped surface \(S_p\); \(S_{\Delta}\) is the set of nodes on the surface \(S_1\), and \(\Omega_{\Delta}\) is the set of internal nodes in the domain \(\Omega\). The approximated Eq. (1.3) is as follows:

\[ \mathcal{P}_{i}^{n+1} - \mathcal{P}_{i-1}^{n+1} = \Delta x_i \lambda_{\text{mix}} \frac{W_{n+1}^{mix}}{2d} \left( W_{n+1}^{mix} W_{n+1}^{mix} \right) (\rho_o \sigma_{n}^{o} + \rho_w \sigma_{n}^{w}). \]

(6)