Dynamical Mean-Field Theory for the Normal Phase of the Attractive Hubbard Model

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We analyze the normal phase of the attractive Hubbard model within dynamical mean-field theory. We present results for the pair-density, the spin-susceptibility, the specific heat, the momentum distribution, and for the quasiparticle weight. At weak coupling the low-temperature behavior of all quantities is consistent with Fermi liquid theory. At strong coupling all electrons are bound in pairs, which leads to a spin gap and removes fermionic quasiparticle excitations. The transition between the Fermi liquid phase and the pair phase takes place at a critical coupling of the order of the band-width and is generally discontinuous at sufficiently low temperature.

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1. INTRODUCTION

The size of Cooper pairs in high-temperature cuprate superconductors is not much bigger than the average distance of conduction electrons in these materials.\textsuperscript{1} This experimental fact has dramatically increased the interest in electronic model systems where attractive interactions can lead to bound electron pairs of arbitrary size, between the BCS-limit of very large Cooper pairs and the opposite Bose limit, where the pairs are smaller than the average particle distance.\textsuperscript{2} Already in 1980 Leggett\textsuperscript{3} pointed out that the superconducting BCS ground state at weak coupling evolves smoothly into a Bose condensate state at strong coupling, as a function of increasing interaction strength. Nozières and Schmitt-Rink\textsuperscript{4} considered the BCS-Bose Crossover at finite temperatures and argued that also the transition temperature $T_c$ between the normal and superconducting (or superfluid) state

\textsuperscript{*}Dedicated to P. Wölfle on the occasion of his 60th birthday
should evolve continuously.

There has also been much interest in possible non-Fermi liquid behavior of the normal phase of electron systems with attractive interactions. The Hubbard model for lattice electrons with a purely local attractive interaction\textsuperscript{5} has become a prototype model in this context. A T-matrix calculation by Frésard \textit{et al.}\textsuperscript{5} for the attractive Hubbard model showed convincingly that Fermi liquid theory governs the normal phase for relatively weak coupling strength even in two dimensions, except for very low density. Only very close to $T_c$ deviations from Fermi liquid behavior due to superconducting fluctuations occur at weak coupling.\textsuperscript{7}

Quantum Monte Carlo (QMC) simulations of the two-dimensional\textsuperscript{8,9} and three-dimensional\textsuperscript{10} attractive Hubbard model have established the formation of a spin gap and a gap in the single-particle excitation spectrum in the normal phase at sufficiently strong coupling. Approximate theories beyond the T-matrix\textsuperscript{11} have produced quite strong pseudogap behavior at intermediate interaction strength in two dimensions.\textsuperscript{12} These results have been related to pseudogap phenomena in underdoped cuprate superconductors.\textsuperscript{13}

In this article we analyze the pair formation and related phenomena in the normal phase of the attractive Hubbard model within the dynamical mean-field theory (DMFT).\textsuperscript{14} This approximation becomes exact in the limit of infinite lattice dimension.\textsuperscript{15} We solve the mean-field equations numerically at finite temperature. The results show that the normal state is a Fermi liquid at weak coupling and a non-Fermi liquid state characterized by bound electron pairs, the absence of fermionic quasi-particles and a spin gap at strong coupling, in qualitative agreement with the QMC studies of finite two- and three-dimensional systems.\textsuperscript{8-10} At very low temperatures the transition between the Fermi liquid and the normal paired state is discontinuous, if the superconducting instability is suppressed. A short account of this work has already appeared.\textsuperscript{16} Here we give more details on the method and present more low temperature data as well as new results for physical quantities not discussed previously. Our analysis is very nicely complemented by a very recent computation of spectral properties of the DMFT solution at zero temperature by Capone \textit{et al.}\textsuperscript{17}

In Sec. 2 we introduce the attractive Hubbard model and discuss some of its elementary properties. In Sec. 3 we motivate and describe the DMFT, with some details on its evaluation. Sec. 4 is dedicated to the presentation and interpretation of results. Most results have been obtained for quarter-filling, but we also present some results for half-filling and filling factor one eighth, to show how the pairing transition depends on density. In Sec. 5 we summarize the results and discuss deficiencies of the DMFT.