A method and application results are described for using mathematical models in statistical analysis and control of the annealing regime of a glass band transported inside a tunnel furnace. The results of a computational experiment estimating the efficiency of the algorithms of the annealing regime control are discussed. The possibility for reducing waste and improving the quality of polished glass annealing is demonstrated.

The introduction of the ISO-9002 quality control system in the production of sheet glass implies the use of statistical methods in controlling the process of polished glass production. We will consider the methods and results of the application of mathematical models for analysis and control of the annealing conditions of a moving glass band in a tunnel furnace.

The method consists of the following stages:

– analysis of the annealing process with the aim of choosing an impulse for evaluation of the quality of annealing and refining the determining factors;
– development of “regime – quality” mathematical models describing the dependence of the glass quality parameters on the annealing temperature regime and the monitored variable input parameters;
– development of an algorithm for adjusting the temperature regime depending on the geometrical dimensions of the band and the line capacity, taking into account the controlled disturbances.

The performance of technological line 1 of an annealing furnace produced by the KNUD company was analyzed. The line capacity is 420 tons/day, the glass band width up to 3300 mm, and the band thickness up to 6 mm. For one month the annealing regime data were registered every 4 h and the glass quality parameters were determined.

The quality of annealing was estimated based on the residual stress value, the flat band curvature, and the percent of glass waste after annealing.

The residual stresses in glass were monitored at five different points across the band width. The measurement data are correlated, and the correlation coefficient is 0.83 – 0.95. This made it possible to estimate the residual stress based on the measurement of stresses in the middle of the band. To improve the accuracy, the residual stress was calculated as the arithmetic mean of five measurement results.

The curvature of the manufactured glass was monitored in samples taken from the left and from the right sides of the glass band. The lateral and longitudinal curvatures of the lower and the upper band surfaces were measured. The measurement results correlate, and the correlation coefficient ranges from 0.60 to 0.89. The generalized parameter of the band curvature after annealing was calculated as the arithmetic mean of 8 sample measurements.

The daily glass waste after annealing was calculated based on the operating time of the glass-crushing machine considering its efficiency.

To refine the list of developed models, the closeness of the correlation between the indicators of the produced glass quality was determined. The matrix of pairwise correlation coefficients is given in Table 1.

The data in Table 1 point to a significant correlation existing between the parameters of glass annealing quality. The negative correlation between the glass waste and the residual stress is evidence of a substantial amount of waste

<table>
<thead>
<tr>
<th>Glass annealing parameters</th>
<th>Residual stress</th>
<th>Curvature</th>
<th>Glass waste</th>
</tr>
</thead>
<tbody>
<tr>
<td>Residual stress</td>
<td>1</td>
<td>-0.48</td>
<td>-0.32</td>
</tr>
<tr>
<td>Curvature</td>
<td>-0.48</td>
<td>1</td>
<td>0.25</td>
</tr>
<tr>
<td>Glass waste</td>
<td>-0.32</td>
<td>0.25</td>
<td>1</td>
</tr>
</tbody>
</table>
generated in producing thin glass, and the positive correlation between the waste and the curvature indicates an increase in the glass waste resulting from high-curvature glasses. The negative correlation coefficient between the curvature and the residual stress reflects the greater curvature in thin glass, which have lower residual stresses. The signs of the correlation coefficients correctly characterize the relationships between the individual parameters of glass annealing quality. As the coefficients of correlation between individual quality parameters are low in modulus, they served as the basis for developing all models describing the quality of annealing.

Depending on the geometrical dimensions and the chemical compositions of glass, a specific optimum annealing regime is selected for each type of glass. The present study considers the case where the chemical composition of glass is relatively stable, and its variations are neglected. The annealing regime depends on the temperature variations in the tunnel furnace and the velocity of the band motion. The temperature inside the annealing furnace is monitored in zones A, B, C, and D along the tunnel. The temperature variations in each zone are registered at five points across the band width. To choose an impulse characterizing the annealing temperature conditions, the temperature variations along the furnace axis measured at various points across the band were subjected to statistical analysis (Fig. 1).

A confidence interval for the mean temperature in different zones was constructed for each curve. The averaged curves of temperature variations along the annealing furnace at different cross-sections across the glass band fall within the overlapping confidence intervals. This makes it possible to describe the variations in the mean temperature by a linear dependence of the type

$$\theta = a_0 - a_1 x, \tag{1}$$

where $a_0$ is the free coefficient of the equation; $a_1$ is the coefficient of the straight line slope; $x$ is the coordinate of the position of the thermocouple in different furnace zones along the longitudinal axis.

Formally the free coefficient corresponds to the glass band temperature at the entrance to the annealing furnace ($x = 0$), which should depend on the temperature of the free end of the glass band at the exit from the float tank. The closeness of the correlation between $a_0$ and the temperature of the exit end of the band is estimated by a correlation coefficient equal to 0.07, which makes it possible to neglect this dependence. The coefficient of the straight line slope characterizes the intensity of temperature variation along the longitudinal axis of the annealing furnace, which determines the process of annealing of the moving glass band. The effect of $a_0$ and $a_1$ on the quality of annealing is characterized by the correlation coefficients shown in Table 2.

As can be seen, a significant correlation exists only between $a_0$ and the glass curvature, as well as between $a_1$ and residual stress. The effect of the coefficients on other parameters can be neglected due to its low significance.

To select the factors affecting glass annealing, the following monitored variables that correlate with the glass quality parameters were considered: the temperature of tin in the float tank at span 15, the temperature of the glass band end at the exit from the float tank, the glass density, the production line efficiency, and the thickness and output rate of the produced glass band. Due to the analytic dependence of the efficiency on the band parameters and output rate, the efficiency was excluded from the number of analyzed factors. The glass thickness and the output rate were left for analysis. The temperature of the exit end of the band significantly depends on (correlates with) the temperature of the tin at span 15. The pair correlation coefficient for these temperatures is 0.72, which makes it possible to consider only one temperature, i.e., the temperature of the end of the band at the exit from the float tank.

Thus, five factors were selected for the construction of regression models: the temperature of the glass band end at the exit from the float tank, the glass density, the band thickness, the output rate, and the parameters characterizing the temperature distribution along the annealing furnace ($a_0$ and $a_1$).

In developing regression models of the annealing process, the conditions of glass melting and the glass band formation over the tin melt, as well as the chemical composition of the glass, were assumed to be constant. These assumptions simplify the construction of models but affect their precision. The justifiability of these assumptions was substantiated by subsequent studies.

The regression models were constructed from a sampling of 180 experiments reflecting the annealing furnace performance during one month. The structure of the models was refined using the procedure of consecutive regression analysis. At first, all the above listed variables were included in the original structure, and then the insignificant variables were consecutively eliminated using the Student $t$-criterion. The elimination continued, while the multiple correlation coeffi-

**TABLE 2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Residual stress</th>
<th>Curvature</th>
<th>Glass waste</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a_0$</td>
<td>0.03</td>
<td>0.13</td>
<td>0.09</td>
</tr>
<tr>
<td>$a_1$</td>
<td>−0.27</td>
<td>0.07</td>
<td>−0.02</td>
</tr>
</tbody>
</table>