STABILITY OF A CIRCULAR SANDWICH RING
UNDER AN AXIALLY SYMMETRIC TEMPERATURE FIELD
INHOMOGENEOUS ACROSS THE THICKNESS

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The linearized problems on the stability of a circular sandwich ring of symmetric structure under an axially symmetric temperature field inhomogeneous across the core thickness are stated and analytical solutions to them are given. The first problem deals with the mixed flexural buckling form (BF) of the ring as a whole, realized as a result of buckling in one of the load-carrying layers due to formation of precritical pressure stresses in the layer. The second problem considers purely shear BFs when one load-carrying layer is rotated relative to the other. The deformation processes for the load-carrying layers are described by the Kirchhoff–Love model, and for the core of arbitrary thickness — by two models, namely by the equations of the plane problem of elasticity theory and by the model of a transversely soft layer of arbitrary thickness (the same equations simplified by the assumption of zero circumferential normal stresses). Within the frames of the first model adopted for the core, the shear BF is theoretically possible but practically unrealizable, since the mixed flexural BF arises earlier than the shear BF.

Statement of the Problem

Let us consider a hollow cylindrical body in the form of a circular ring placed in an axisymmetric stationary temperature field inhomogeneous across the ring thickness. The self-balanced stressed state arising under the action of such a temperature field can cause buckling of the ring material only according to the local form, with surface buckling in the zone of formation of circumferential compression stresses.

It is evident that the statement of the problem on such a buckling form for the ring is generally possible only within the framework of relevant linearized equations of elasticity theory. In their reduction to equations of a smaller dimensionality (to the equations of the theory of curvilinear rods or shells), we must provide the required degree of accuracy, at least by taking into account the ring deformations in the radial direction.

In the case where the ring material is homogeneous across the thickness, the statement and solution of the above problem is evidently of little practical interest. This problem may prove to be more important for a ring inhomogeneous across its thickness, especially for a sandwich one, whose soft middle layer, named the core, frequently serves as a heat insulator in structures and has, as a rule, small coefficients of thermal expansion compared with those of the material of rigid load-carrying lay-
ers. In a temperature field inhomogeneous across the ring thickness, circumferential tangential forces equal in magnitude but opposite in sign arise in the load-carrying layers if the core is described as a transversely soft layer. Under such a stress state, a mixed flexural buckling form (BF) similar in character to that for a sandwich beam in pure bending [1] can be expected for the ring.

For a thin sandwich ring, the BF described can be examined within the framework of the refined equations given in [2, 3], which are constructed for shells of general type under arbitrary thermal and force actions. However, in this study, the corresponding problem for the core will be stated using two more exact (compared with that adopted in [2]) models, namely the nonsimplified equations of the plane problem of thermoelasticity theory and the same equations but simplified by the assumption of zero circumferential normal stresses. The latter model allows passage to the model of a thin transversely soft layer [2] provided the change in the metric across the core thickness is neglected.

To simplify the solution of this problem, we assume that the structure of the sandwich package is symmetric about the midsurface of the core and the load-carrying layers of the ring are thin.

1. Precritical Stress-Strain State

Let us refer a sandwich ring with the thicknesses $2t$ and $2h$ of the load-carrying layers and the core, respectively, to the radial coordinate $z$ along the normal to the midsurface of the core, $\sigma$, and the circumferential coordinate $\theta$. Let $E$, $v$, $E_3$, and $\alpha_3$ be the elastic moduli and the coefficients of linear thermal expansion of the load-carrying layers and the core, $T_1$ and $T_2$ the temperature increments in the outer layers, which are assumed to be small due to the smallness of $2t$, $T_3 = T_3(z)$ the unknown temperature in the core, $w_0^{(k)} = \text{const}$ ($k = 1$ refers to the lower and $k = 2$ to the upper layer), and $w_0 = w_0(z)$ the unknown radial displacements (deflections) of the load-carrying layers and the core.

If, instead of $z$, we introduce a dimensionless radial coordinate $\rho = 1 + z/R$, within the framework of the uncoupled stationary problem of thermoelasticity, the temperature $T_3$, the circumferential tangential forces in the load-carrying layers, $T_2^{0(k)}$, and the radial and circumferential stresses $\sigma_{33}^0$ and $\sigma_{22}^0$ are determined by the equations of equilibrium

\[
T_2^{0(1)} - R\sigma_{33}^0(\rho_1) = 0, \quad T_2^{0(2)} + R\sigma_{33}^0(\rho_2) = 0,
\]

\[
\frac{d\sigma_{33}^0}{d\rho} + \frac{\sigma_{33}^0 - \sigma_{22}^0}{\rho} = 0,
\]

and heat conduction

\[
\frac{d}{d\rho}\left(\rho \frac{dT_3}{d\rho}\right) = 0,
\]

where $\rho_k = 1 - \delta_k h_0$, $h_0 = h/R$, $\delta_{(1)} = -\delta_{(2)} = 1$, and $\rho_1 \leq \rho \leq \rho_2$.

The solution of heat conduction equation (1.3) with $T_k = T_3(\rho_k)$ ($k = 1, 2$) is

\[
T_3 = T_0 + T \ln \rho,
\]

where

\[
T = \frac{T_2 - T_1}{\ln \rho_2/\rho_1}, \quad T_0 = \frac{T_1 \ln \rho_2 - T_2 \ln \rho_1}{\ln \rho_2/\rho_1}.
\]

Systems (1.1) and (1.2) will be solved for two variants of the models accepted for describing the behavior of the core: — a model of a transversely soft layer characterized by the absence of a circumferential stress in the core ($\sigma_{22}^0 = 0$) and