A model is considered of an S-ALOHA channel with nonpersistent subscribers. It is proved that in this case the optimum strategy for repetitions is of a nonsteady-state nature and consists of an equal-probability choice of the repetition segments during the time-out interval.

In communication networks with insecure channels, such as second- and third-generation mobile cellular communication networks and also satellite communication systems of mobile subscribers, an important part is played by the process of the access of the mobile station to the serving base station (for cellular communication systems) [1, 2]. As a rule, the implementation of this procedure of multiple access in the initial stage of organizing communications is based on the use of a protocol of the segmented ALOHA type in specially selected (inquiry–call) channels (PRACH, Physical Random Access Channels) with a persistent strategy for obtaining access [3]. The concept of “access attempt” is used in the mechanism of establishing communication of such a protocol. A successful access attempt implies a procedure of transmission along a given channel of one short message, a packet (an inquiry about the availability of a working channel) and the reception of a signal confirming its reception by the base station. Attempts continue until such time that the confirmation of reception is obtained.

Let us consider a segmented ALOHA channel (S-ALOHA) with $N$ subscribers. The subscribers may be either in the state of the primary generation of packets or in the regime of their repeated transmission (a conflict situation). In the primary generation state, each new packet is transmitted to a subscriber in the next time segment (window) of the channel with a probability of $p_0$, while in the repeated transmission regime it is transmitted with a probability $p_r$. In real systems one usually has that $p_r > p_0$, and so we shall adhere to this inequality.

The state of a channel $n = \{0, 1, 2, ..., N\}$ is taken to be the number of subscribers who are in the repeated transmission regime. The matrix of the probabilities of transitions between the states, $P = \|p_{nm}\|$, where $p_{nm} = P[x_t = m | x_{t-1} = n]$ is given by the expression

$$
p_{nm} = \begin{cases} 
0, & m \leq n - 2; \\
(1 - p_0)^{N-n} n p_r (1 - p_r)^{n-1}, & m = n - 1; \\
(1 - p_0)^{N-n} [1 - np_r (1 - p_r)^{n-1}] + (N-n)p_0(1-p_0)^{N-n-1}(1-p_r)^n, & m = n; \\
(N-n)p_0(1-p_0)^{N-n-1}[1 -(1-p_r)^n], & m = n + 1; \\
C_{N-n}^m p_0^m (1-p_0)^{N-n}, & m \geq n + 2.
\end{cases}
$$

The strategy of repeated transmissions is the rule of the selection of a repeated transmission probability $p_r$ in the next segment of the channel as the process of its functioning is developing. If the repeated transmission probability is indepen-
dent of time and of the state of the channel \((p_r = \text{const})\), then such a strategy is said to be steady-state and uniform. For such strategies, in view of the ergodicity and regularity of the Markov chain, a steady-state vector \(\Pi = (\pi_0, \pi_1, ..., \pi_N)\) of the probabilities of the states of the channel exists which satisfies the equations

\[
\pi_m = \sum_{n=0}^{N} \pi_n p_{nm}, \quad m = \overline{0, N}; \quad \sum_{n=0}^{N} \pi_n = 1.
\]  

(2)

The quality indicators of the strategy are the transmissivity of the channel (in packets per segment) and the average delay time of a packet (in segments), determined by the expressions

\[
S = \sum_{n=0}^{N} \pi_n a_n, \quad T = \pi / s,
\]

where \(a_n\) is the transmissivity of the channel in the \(n\)th state, \(a_n = (1 - p_0)^N - n - 1 \cdot (N - n) p_0 (1 - p_r) + n p_r (1 - p_0)\); \(\bar{n}\) is the average number of subscribers in the regime of repeated transmission of a packet, \(\bar{n} = \sum_{n=1}^{N} n \pi_n\).

The optimal steady-state uniform repetition strategy is considered to be such a value \(p_r^*\) which ensures the maximum transmissivity of the channel. It is well known [4] that the optimal strategy corresponds to the maximum value of \(p_r\) for which a unimodal probability distribution \(\pi_n\) \((n = 0, N)\) is maintained.

A subscriber who has a limited time being in a state of repeated transmission will be considered to be nonpersistent. The maximum permissible time (in segments) for effecting repetitions is called the time-out. After the expiry of the time-out a subscriber is refused transmission of the current packet (he obtains a failure to connect) and is forced to transfer to the primary generation regime. The probability of a failure to connect \(q_{\text{to}}\) evidently depends on the parameters of the S-ALOHA channel (and in particular on the chosen repetition strategy), the time-out duration \(\tau\), and the current state \(n\) of the channel. Under these conditions, the matrix of transition probabilities \(P^{(H)} = \|P_{nm}^{(H)}\|\) expressed in terms of system (1) will be of the form

\[
P_{nm}^{(H)} = \begin{cases} 
p_{nm}, & n = 0; \\
\min(N-m,n) \sum_{i=n-m}^{i=1} \binom{n}{i} q_{\text{to}}^i (1-q_{\text{to}})^{n-i} p_{ni+m}, & m < n \lor n > 0; \\
\min(N-m,n) \sum_{i=n-m}^{i=1} \binom{n}{i} q_{\text{to}}^i (1-q_{\text{to}})^{n-i} p_{ni+m}, & m = n \lor n > 0; \\
\min(N-n,m) \sum_{i=m-n}^{i=1} \binom{n}{i} q_{\text{to}}^i (1-q_{\text{to}})^{n-i+1} p_{nn+i}, & m > n \lor n > 0.
\end{cases}
\]

(3)

An indicator of the quality of such a channel will be considered to be the probability of its accessibility to a subscriber:

\[
P_{\text{tr}} = 1 - Q,
\]

(4)

where \(Q\) is the probability of failure to connect, defined as

\[
Q = \sum_{n=1}^{N} \pi_n^{(H)} q_{\text{to}}.
\]

(5)

1192