INTRODUCTION

Nuclear-pumped lasers are subjected to intensive studies in Russia and in the United States [1–3]. The principle of nuclear pumping of lasers involves the inverse population of laser levels by way of irradiating a gas by fission fragments from thin layers of uranium applied to the inner surface of a laser cuvette (see Fig. 1). Ultimately, the bulk of the energy of fission fragments transforms to the thermal energy of gas. As a result of heat transfer to the wall, considerable differences in the gas temperature and density arise in the wall regions of the cuvette, where the laser beams bend strongly [4, 5]. When the gas is pumped through the laser channel [6, 7], a thermal boundary layer is formed at the channel walls. The qualitative singularity of such a layer is defined by internal heat release (energy contribution by the fission fragments) at a gas velocity that is much lower than the velocity of sound; in a boundary layer [8, 9] at transonic or supersonic velocities, the source of heat is provided by the dissipation of energy due to the effect of viscous forces. A similar problem was treated for a boundary layer with heat release due to the condensation of vapor at a supersonic velocity [10]. The wall layer in nuclear-pumped flowing lasers was previously treated [11] as purely thermal, disregarding the viscosity (within the model of core flow of gas). In numerical calculations of the gas dynamics of a flowing laser [6], the boundary layer was included but was not treated separately. The temperature of the channel walls in [6, 11] was taken to be uniform.

This paper deals with the investigation of the wall layer in flat channels of nuclear-pumped flowing lasers with due regard for the viscosity and internal heat release for a nonuniform temperature of the channels walls.

GASDYNAMIC MODEL

We will treat the steady-state mode of pumping an inert gas (He, Ar) through a flat channel [7] to whose walls thin layers of uranium irradiating the gas were applied (Fig. 1). The \( x \)-axis is directed along the gas flow, and the \( y \)-axis, across the flow (reckoned from the bottom wall of the channel). We will assume that the channel length along the gas flow \( b \sim 10 \text{ cm} \), the channel width \( d \sim 2 \text{ cm} \), the gas pressure \( P_0 \sim 1 \text{ atm} \), the gas temperature \( T_0 \sim 300 \text{ K} \), and the gas pumping rate \( U_0 = 10 \text{ to } 100 \text{ m/s} \) (the subscript 0 indicates the initial values at the channel inlet). The gas flow is assumed to be laminar both in the main flow and in the boundary layer. Indeed, the critical Reynolds number for the thickness \( \delta \) of hydrodynamic boundary layer is estimated [8] at

\[
\text{Re}_\delta^* = 3000, \quad \text{Re}_\delta = \frac{U_0 \delta}{\nu}, \quad \delta = \sqrt[3]{\frac{\nu x}{U_0}} \leq \sqrt[3]{\frac{\nu b}{U_0}},
\]

where \( \nu \sim 10^{-4} \text{ m}^2/\text{s} \) is the kinematic viscosity coefficient. In the case being treated, \( \delta \sim 0.03 \text{ to } 0.1 \text{ cm} \ll d \) and \( \text{Re}_\delta \sim 100 \text{ to } 300 \ll \text{Re}_\delta^* \). A number of factors are disregarded here. When the gas absorbs the energy of fission fragments, it expands and accelerates along the
flow, which is the stabilization factor of the boundary layer [8]. On the other hand, the gas density drop across the layer promotes the thermogravitational convection. The effect of this factor on the stability of the boundary layer is characterized by the Richardson number [8],

\[ Ri = \frac{g \Delta p}{\rho U_0^2} \left( \frac{du}{dy} \right)^2, \]

where \( g = 9.8 \text{ m/s}^2 \), \( \rho \) is the gas density, \( \Delta p \) is the density drop, and \( u \) is the longitudinal velocity of gas. With so small a value of \( Ri \), the thermogravitational effects have little influence on the critical Reynolds number. In the main flow of gas, the characteristic time of pumping (\( \tau = b/U_0 \sim 10^{-2} \) to \( 10^{-3} \) s) is too short for the thermogravitational convection to develop.

The equations of the thermal boundary layer have the form [8, 9]

\[ \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho w) = 0, \]

\[ \rho \frac{\partial u}{\partial x} + \rho \frac{\partial w}{\partial y} = \frac{\partial}{\partial y} \left( \frac{\mu}{c_p} \frac{\partial T}{\partial y} \right) - \frac{dP}{dx}, \quad (1) \]

\[ \rho \frac{\partial T}{\partial x} + \rho \frac{\partial T}{\partial y} = \frac{\partial}{\partial y} \left( \frac{\lambda}{c_p} \frac{\partial T}{\partial y} \right) + \frac{1}{c_p} \left( \delta Q/\delta V \delta t \right). \quad (2) \]

Here, \( \delta Q \) is the heat released by fission fragments in the gas volume \( \delta V \) during the time \( \delta t \). The Mach number \( M \) is rather small (\( M < 0.01 \)); therefore, in Eq. (2) of energy transfer, the dissipation of energy due to viscous forces may be ignored, and the gas pressure may be taken to be uniform. The gas is assumed to be ideal,

\[ P = \gamma - 1 \rho c_p T = P_0 = \frac{\gamma - 1}{\gamma} \rho_0 c_p T_0, \quad (3) \]

where \( \gamma \) is the ratio of heat at constant pressure to heat at constant volume (\( \gamma = 5/3 \)). The degree of ionization of gas is low (\( \sim 10^{-7} \)), so that the effect of the plasma composition on the thermophysical properties of gas may be ignored.

The energy contribution of fission fragments may be represented in the form [5]

\[ \frac{\delta Q}{\delta V \delta t} = \frac{\Theta P_0}{(\gamma - 1) \rho_0} \frac{U_0}{b} \frac{F}{\rho_0 c_p T_0 F} \frac{U_0}{b}. \quad (4) \]

Here, the energy contribution \( \Theta \) is the ratio of energy, contributed by fission fragments to a cuvette with a gas of uniform density \( \rho_0 \) during the characteristic time \( \tau = b/U_0 \), to the internal energy of this gas (in practice, \( \Theta \leq 0.5 \)); and \( F(x, y) \sim 1 \) is the dimensionless distribution function of energy contribution, defined by the cuvette geometry and gas density. In the case of low values of \( \Theta \) within a narrow boundary layer, one can assume that \( F(x, y) = \Phi(x, 0) = \text{const} = \Phi^* \leq 1 \).

**APPROXIMATION OF SMALL ENERGY CONTRIBUTIONS**

In the problem being treated, it is reasonable to change from the transverse coordinate \( y \) to the self-similar variable \( \eta(x, y) \) using Dorodnitsyn’s variable \( Y [12] \),

\[ \rho_0 dY = \rho(x, y) dy, \]

\[ \eta(x, y) = \frac{U(x)}{\sqrt{v_0 x}}, \quad (5) \]

where \( U(x) \) is the gas velocity in the main flow, and \( U(0) = U_0 \). On introducing the dimensionless stream function \( f(x, \eta) \) and the relative temperature \( \vartheta(x, \eta) \),

\[ u(x, y) = U(x) \frac{\partial f(x, \eta)}{\partial \eta}, \quad (6) \]

and assuming that

\[ \mu(T) = \mu_0 \left( \frac{T}{T_0} \right)^{1-\alpha}, \quad \sigma = \frac{1}{4}, \quad (7) \]

\[ \lambda(T) = \lambda_0 \left( \frac{T}{T_0} \right)^{1-\nu}, \quad \varepsilon = \frac{1}{4}, \quad (8) \]

where \( \sigma \) and \( \varepsilon \) are small parameters, Eqs. (1)–(4) may be rewritten as

\[ \frac{\partial}{\partial \eta} \left( \vartheta \frac{\partial f}{\partial \eta} \right) + \frac{1}{2} \left[ 1 + \frac{\vartheta}{U^*} \right] f \vartheta, \quad (9) \]

\[ + \frac{x}{U^*} \left[ \frac{T}{U} - (f^*)^2 \right] = x (f^* f^* - f^* f^*), \quad (10) \]

\[ \frac{1}{Pr \vartheta} \left( \vartheta \frac{\partial f}{\partial \eta} \right) + \vartheta \left[ 1 + \frac{x}{U^*} \right] f \vartheta, \quad (11) \]

\[ + \frac{\vartheta}{\gamma} \frac{x}{b} \frac{U_0}{\vartheta} = x (f^* f^* - f^* f^*), \quad (12) \]

\[ \varphi = \frac{\rho \mu}{\rho_0 \mu_0} = \left( \frac{T}{T_0} \right)^{1-\sigma}, \quad \psi = \frac{\rho \lambda}{\rho_0 \lambda_0} = \left( \frac{T}{T_0} \right)^{1-\varepsilon}, \quad (13) \]

\[ Pr = \frac{v_0}{\lambda_0} \rho_0 c_p. \quad (14) \]