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On the Hardness of the Quadratic Assignment Problem with Metaheuristics

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Abstract

Meta-heuristics are a powerful way to approximately solve hard combinatorial optimization problems. However, for a problem, the quality of results can vary considerably from one instance to another. Understanding such a behaviour is important from a theoretical point of view, but also has practical applications such as for the generation of instances during the evaluation stage of a heuristic.

In this paper we propose a new complexity measure for the Quadratic Assignment Problem in the context of metaheuristics based on local search, e.g. simulated annealing. We show how the ruggedness coefficient previously introduced by the authors, in conjunction with the well known concept of dominance, provides important features of the search space explored during a local search algorithm, and gives a rather precise idea of the complexity of an instance for these heuristics. We comment previous experimental studies concerning tabu search methods and genetic algorithms with local search in the light of our complexity measure. New computational results with simulated annealing and taboo search are presented.

Key Words: quadratic assignment problem, local search, complexity measures

1. Introduction

Given two $n \times n$ matrices $F = (f_{ij})$ and $D = (d_{ij})$, the Quadratic Assignment Problem (QAP) can be stated as follows:

$$\min_{\pi} \sum_{ij} f_{ij} d_{\pi(i)\pi(j)}.$$

The QAP is known to be NP-hard (Garey and Johnson, 1979), and non approximable (Sahni and Gonzalez, 1976). In the context of location theory, f_{ij} is the flow of materials from plant i to plant j, and d_{ij} represents the distance from location i to location j. The objective is to find an assignment of all plants to locations such that the sum of products distance $\times$ flow is minimized. In the sequel we shall assume that the matrices F and D are symmetric with a null diagonal, and the cost function to be minimized is written $C(\pi) = \frac{1}{2} \sum_{i,j=1}^{n} f_{ij} d_{\pi(i)\pi(j)}$.

The QAP is a computationally very hard combinatorial optimization problem, and instances with a size greater than 30 are practically not solvable to optimality. Fortunately it has been noticed that heuristics give remarkably good results, and specially those based on

local search, e.g. simulated annealing and tabu search. For example, Maniezzo, Dorigo, and Colorni have concluded in Dorigo, Maniezzo, and Colorni (1995): "[...] reflects a QAP property which was not yet put into evidence in the literature: any approach based on local search is bound to be very effective as a heuristic for QAP", and Connolly writes in Connolly (1990): "Simulated annealing is an extremely efficient heuristic for the QAP". These methods use the 2-exchange neighborhood which consists in exchanging the locations of two plants. More formally, given a permutation $\pi = (\pi(1), \ldots, \pi(i), \ldots, \pi(j), \ldots, \pi(n))$, its neighbors are the $n(n-1)/2$ permutations of the form $\pi' = (\pi(1), \ldots, \pi(j), \ldots, \pi(i), \ldots, \pi(n))$ for $1 \le i < j \le n$, obtained from π by a swap.

In Angel and Zissimopoulos (1998) the authors have presented a theoretical performance guarantee result for a basic local search procedure, nonetheless the quality of solutions obtained with metaheuristics can vary considerably from one instance to another. This leads us to the notion of a complexity measure for the QAP. Generally speaking, by this we mean that given an algorithm and an instance, to be able to characterize the difficulty of that instance, and to say if the algorithm is adapted or not for it. Of course, the interpretation of the difficulty is function of the method used, and the aim pursued. For example, it can be a prediction of the computational time needed to solve the problem by an exact algorithm, or an indication of how far one can be from the optimal solution when an approximate method is used.

This complexity measure has important applications. It can be useful for choosing among several methods of resolution the more adapted for the instance considered, and also in the evaluation stage of an algorithm. Indeed, when one tests it, it is important to ensure that instances that are generated are well distributed in the "complexity space" of the problem. One should have both easy and hard instances.

The next of this paper is organized as follows: in Section 2 we discuss the concept of flow dominance and review some previous works on the hardness of QAP instances. We generalize the concept of dominance in Section 3. In Section 4 we comment the notion of the ruggedness of a landscape and its link with local search, then we discuss about our two previously defined autocorrelation and ruggedness coefficients. In Section 5, a precise characterization of the difficulty of a QAP instance for local search based heuristics is presented. The Section 6 is devoted to experimental evaluations.

2. The flow dominance

The standard complexity measure for the QAP is the *flow dominance*, which was introduced by Vollmann and Buffa (1966). It measures to what extent the flow matrix F shows "dominant" flow patterns. A possible definition of the flow dominance is given by Merz and Freisleben (1997) and Herroelen and Van Gils (1985):

$$fd(F) = 100\frac{\sigma}{\mu}, \quad \text{with}$$

$$\mu = \frac{1}{n^2}\sum_{i=1}^{n}\sum_{j=1}^{n} f_{ij}, \quad \text{the mean and}$$

$$\sigma = \sqrt{\frac{1}{n^2-1}\sum_{i=1}^{n}\sum_{j=1}^{n}(f_{ij}-\mu)^2},$$