ON $\theta$-PREIRRESOLUTE FUNCTIONS

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Abstract. We introduce a new class of functions called $\theta$-preirresolute functions which is contained in the class of quasi preirresolute functions and contains the class of preirresolute functions. It is shown that $p$-closedness and $\beta$-connectedness are preserved by $\theta$-preirresolute surjections.

1. Introduction

A subset $A$ of a topological space $X$ is called preopen [12] or nearly open [20] if $A \subseteq \text{Int} \left( \text{Cl}(A) \right)$. A function $f : X \to Y$ is called precontinuous [12] if the preimage $f^{-1}(V)$ of each open set $V$ of $Y$ is preopen in $X$. Precontinuity was called near continuity by Prik [20] and also called almost continuity by Frolik [7] and Hussain [8]. In 1985, Reilly and Vamanmurthy [21] defined a function to be preirresolute if the preimage of every preopen set is preopen. Recently, Pal and Bhattacharyya [17] have introduced strong preirresoluteness and quasi preirresoluteness, as strong and weak form of preirresoluteness, parallel to strong irresoluteness and quasi irresoluteness due to Di Maio and Noiri [4].

In this paper, we introduce a new class of functions called $\theta$-preirresolute functions which is contained in the class of quasi preirresolute functions and contains the class of preirresolute functions. We investigate $\theta$-preirresolute functions and obtain several improvements of results established by Pal and Bhattacharyya [17]. It is also shown that $p$-closed spaces [5] and $\beta$-connected spaces [19] are preserved by $\theta$-preirresolute surjections.

2. Preliminaries

Throughout the present paper, by $(X, \tau)$ and $(Y, \sigma)$ (or simply $X$ and $Y$) we denote topological spaces. Let $S$ be a subset of $X$. We denote the interior and the closure of a set $S$ by $\text{Int}(S)$ and $\text{Cl}(S)$, respectively. A subset $S$ is

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said to be preopen [12] (resp. semi-open [10], \(\alpha\)-open [14]) if \(S \subset \text{Int} (\text{Cl}(S))\) (resp. \(S \subset \text{Cl}(\text{Int}(S))\), \(S \subset \text{Int}(\text{Cl}(\text{Int}(S)))\)). The complement of a preopen set is called \textit{preclosed}. The intersection of all preclosed sets containing \(S\) is called the \textit{preclosure} [6] of \(S\) and is denoted by \(\text{pCl}(S)\). The \textit{preinterior} of \(S\) is defined by the union of all preopen sets contained in \(S\) and is denoted by \(\text{pInt}(S)\). The family of all preopen sets of \(X\) is denoted by \(\text{PO}(X)\). We set \(\text{PO}(X, x) = \{U : x \in U \text{ and } U \in \text{PO}(X)\}\). A point \(x\) of \(X\) is called a \textit{pre-\(\theta\)-cluster point} of \(S\) if \(\text{pCl}(U) \cap S \neq \emptyset\) for every preopen set \(U\) of \(X\) containing \(x\). The set of all pre-\(\theta\)-cluster points of \(S\) is called the \textit{pre-\(\theta\)-closure} of \(S\) and is denoted by \(\text{pCl}_{\theta}(S)\). A subset \(S\) is said to be \textit{pre-\(\theta\)-closed} [17] if \(S = \text{pCl}_{\theta}(S)\). The complement of a pre-\(\theta\)-closed set is said to be \textit{pre-\(\theta\)-open}.

**Definition 2.1.** Let \(X\) and \(Y\) be topological spaces and \(S\) a subset of the product space \(X \times Y\). A subset \(S\) is said to be \textit{strongly pre-\(\theta\)-closed} (resp. \textit{strongly pre-\(\theta\)-preclosed}) [17] if for each point \((x, y) \in (X \times Y) - S\), there exist \(U \in \text{PO}(X, x)\) and \(V \in \text{PO}(Y, y)\) such that \((\text{pCl}(U)) \times \text{pCl}(V) \cap S = \emptyset\) (resp. \([U \times \text{pCl}(V) \cap S = \emptyset\)).

**Remark 2.1.** By Lemma 7.10 of [17], we have the following diagram:

\[
\text{strongly pre-\(\theta\)-closed} \Rightarrow \text{pre-\(\theta\)-closed} \\
\downarrow \quad \downarrow \\
\text{strongly pre-\(\theta\)-preclosed} \Rightarrow \text{pre-\(\theta\)-closed}
\]

**Definition 2.2.** A function \(f : X \to Y\) is said to be \textit{preirresolute} [21] if \(f^{-1}(V) \in \text{PO}(X)\) for every \(V \in \text{PO}(Y)\).

**Definition 2.3.** A function \(f : X \to Y\) is said to be \textit{strongly preirresolute} (resp. \textit{quasi preirresolute}) [17] if for each \(x \in X\) and each \(V \in \text{PO}(Y, f(x))\), there exists \(U \in \text{PO}(X, x)\) such that \(f(\text{pCl}(U)) \subseteq V\) (resp. \(f(U) \subseteq \text{pCl}(V)\)).

**Definition 2.4.** A function \(f : X \to Y\) is said to be \(\theta\)-preirresolute if for each \(x \in X\) and each \(V \in \text{PO}(Y, f(x))\), there exists \(U \in \text{PO}(X, x)\) such that \(f(\text{pCl}(U)) \subseteq \text{pCl}(V)\).

**Remark 2.2.** By the above definitions and proof of Theorem 3.3, we have the following implications and none of these implications is reversible by Example 5.4 of [17], the following example and Example 6.2 (below):

\[
\text{strongly preirresolute} \Rightarrow \text{preirresolute} \\
\Rightarrow \text{\(\theta\)-preirresolute} \Rightarrow \text{quasi preirresolute}
\]

**Example 2.1.** Let \(X = \{a, b, c\}\), \(\tau = \{\emptyset, X, \{a\}, \{a, b\}\}\) and \(\sigma = \{\emptyset, X, \{a\}, \{a, c\}\}\). We define a function \(f : (X, \tau) \to (X, \sigma)\) by \(f(a) = b\), \(f(b) = c\).