A COMPUTATIONAL METHOD FOR INTERVAL MIXED
VARIABLE ENERGY MATRICES IN
PRECISE INTEGRATION*

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Abstract: To solve the Riccati equation of LQ control problem, the computation of interval
mixed variable energy matrices is the first step. Taylor expansion can be used to compute
the matrices. According to the analogy between structural mechanics and optimal control
and the mechanical implication of the matrices, a computational method using state tran-
sition matrix of differential equation was presented. Numerical examples are provided to show
the effectiveness of the present approach.

Key words: precise integration; Riccati equation; optimal control

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1 Introduction and Problem

The solution of the Riccati equation is one of the key steps in the process of optimal control.
Based on theories of structural mechanics and optimal control, precise integration method was
developed and can be used to solve the Riccati equation and algebraic equation accurately with
high efficiency. For example, in the LQ control, the system equation can be expressed as¹¹
\[ \dot{x} = Ax + Bu, \quad x(0) = x_0, \]
(1)
where \( x \) is the state vector of \( n \) dimensions, \( u \) is the input vector of \( p \) dimensions; \( A, B \) are
steady matrices of corresponding dimensions, where \( (A, B) \) can be controlled. The performance
index is
\[ J = \frac{1}{2} x_1^T S_1 x_1 + \frac{1}{2} \int_0^T (x^T Q x + u^T R u) \, dt, \]
(2)

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where $x_f = x(t_f)$, $Q$ and $s_f$ are positive semi-definite matrices, $R$ is a positive definite matrix.

To attain the optimal control, $u(t)$ is selected so that the function $J = J(u)$ reaches minimum. The optimal control with satisfying condition can be expressed as follows:

$$
u(t) = - R^{-1} B^T \lambda(t),$$

$$\lambda(t) = P(t) x(t),$$

where $\lambda(t)$ is the co-state variable, and $P(t)$ is the solution of the following Riccati differential matrix equations

$$\dot{P}(t) = - P(t) A - A^T P(t) + P(t) BR^{-1} B^T P(t) - Q,$$

$$P(t_f) = s_f.$$  \(5\)

When solving above-mentioned equations by precise integration method, the mixed variable energy of interval $(t_a, t_b)$ should be defined first\(^{[2]}\). The definitions of state and co-state are $x_a = x(t_a)$, $\lambda_a = \lambda(t_a)$, $x_b = x(t_b)$, $\lambda_b = \lambda(t_b)$ respectively at the $(t_a, t_b)$ moment. The mixed variable energy during the interval $(t_a, t_b)$ is

$$V(x_a, \lambda_b) = \lambda_a^T x_b - \int_{t_a}^{t_b} \left( \lambda^T \dot{x} - \frac{1}{2} x^T Q x - \lambda^T A x + \frac{1}{2} \lambda^T B R^{-1} B^T \lambda \right) dt.$$  \(6\)

Since $V(x_a, \lambda_b)$ is a quadratic equation of $x_a$ and $\lambda_b$, it can be rewritten as

$$V(x_a, \lambda_b) = \lambda_b^T F x_a + \frac{1}{2} X_a^T E X_a - \frac{1}{2} \lambda_b^T G \lambda_b,$$  \(7\)

where $F$, $E$ and $G$ are interval mixed variable energy matrices of $n \times n$ order. They are related to matrices $A$, $BR^{-1} B^T$ and $Q$. They satisfy differential equations\(^{[3, 3]}\):

$$\frac{dF}{d\tau} = Q + A^T E + EA - EBR^{-1} B^T E,$$  \(8\)

$$\frac{dG}{d\tau} = FBR^{-1} B^T F^T,$$  \(9\)

$$\frac{dF}{d\tau} = (A - GQ) F,$$  \(10\)

where $\tau = t_b - t_a$, and during $t_a \rightarrow t_b$, there are

$$E \rightarrow 0, \ G \rightarrow 0, \ F \rightarrow I.$$  \(11\)

If time difference of Eq. (8) is reversed, it turns to Riccati Eq. (5). So the computation of matrix $E$ equals to the solving of Riccati equation.

2 The Computation of Interval Mixed Variable Energy Matrices

To solve mixed variable energy matrices $E$ in differential Eq. (8), the basic interval mixed variable energy should be computed first, and then interval merging formula recursion is processed. Basic interval length is defined as step length $\eta$ of time-displacement integration. It is divided into $m = 2^N$ sections, usually taking $N = 20$, gets $m = 1\,048\,576$, and the corresponding interval