NEW EXPLICIT AND EXACT TRAVELLING WAVE SOLUTIONS
FOR A CLASS OF NONLINEAR EVOLUTION EQUATIONS*

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Abstract: With the help of Mathematica, many travelling for a class of nonlinear evolution equations \( u_t + au_{xx} + bu + cu^2 + du^3 = 0 \) are obtained by using hyperbola function method and WU-elimination method, which include new travelling wave solutions, periodic solutions and kink soliton solutions. Some equations such as Duffing equation, sin-Gordon equation, \( \varphi^4 \) and Klein-Gordon equation are particular cases of the evolution equations. The method can also be applied to other nonlinear equations.

Key words: equation; periodic solution; solitary wave solution

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Introduction

With the rapid development of science and technology, the study kernel of modern science is changed from linear to nonlinear step by step. Many nonlinear science problems can simply and exactly be described by using the mathematical model of nonlinear equation. Up to now, many important physical nonlinear evolution equations are found, such as sin-Gordon equation, KdV equations, Schrodinger equation all possess solitary wave solutions. There exist many methods to seek for the solitary wave solutions, such as inverse scattering method, Hopf-Cole transformation, Miura transformations, Darbour transformation and Backlund transformation\(^{1-5}\), but solving nonlinear equations is still an important task.

In this paper, with the aid of Mathematica, a few of traveling wave solution for a class of nonlinear evolution equations

\[ u_t + au_{xx} + bu + cu^2 + du^3 = 0 \]  

\((a, b, c, d \text{ being constants})\) are obtained by using the idea of hyperbola function method\(^6\) and WU-elimination method\(^7\). Duffing equation, Sin-Gordon equation, \( \varphi^4 \) equation and Klein-
Gordon equation, as the special cases of this equation, also obtain the several corresponding travelling wave solutions.

1 New Travelling Wave Solution and Solitary Wave Solution for Eq. (1)

Firstly, making the travelling waves reduction transformation for Eq. (1)
\[ u(x,t) = \varphi(\xi), \quad \xi = \lambda(x - kt + c_0), \]  
where \( \lambda \) and \( k \) are constants to be determined later, \( c_0 \) is an arbitrary constant.

Substituting (2) into (1) yields an ordinary differential equation for \( \varphi \)
\[ \lambda^2(k^2 + a) \frac{d^2 \varphi}{d\xi^2} + b\varphi + c\varphi^2 + d\varphi^3 = 0. \]  
(3)

Suppose that Eq. (3) has the following formal travelling wave solutions:
\[ \varphi(\xi) = A_1 \sinh w + A_2 \cosh w + A_0. \]  
(4a)

(1) taking the target equation
\[ \frac{dw}{d\xi} = \sinh w, \]  
(4b)

where \( A_0, A_1, A_2 \), are constants to be determined later.

Substituting (4a) and (4b) into (3), with the aid of Mathematica, we have
\[ \lambda^2(k^2 + a) \frac{d^2 \varphi}{d\xi^2} + b\varphi + c\varphi^2 + d\varphi^3 = \lambda^2(k^2 + a)(2A_1 \sinh^3 \varphi + \]
\[ 2A_2 \sinh^2 w \cosh w + A_1 \sinh w + b(A_1 \sinh w + A_2 \cosh w + A_0) + c(A_1^2 + A_2^2) \sinh^2 w + 2cA_1 A_2 \sinh w \cosh w + 2cA_0 A_1 \sinh w + 2cA_0 A_2 \cosh w + \]
\[ c(A_0^2 + A_1^2) + d(A_1^3 + 3A_1 A_2^2) \sinh^3 w + d(A_1^3 + 3A_2^3 A_1) \sinh^2 w \cosh w + \]
\[ d(3A_0 A_1^2 + 3A_0 A_2^3) \sinh^2 w + d(A_1^2 + 3A_0^2 A_2) \cosh w + \]
\[ 6dA_0 A_1 A_2 \sinh w \cosh w + d(3A_1 A_2^2 + 3A_0^2 A_1) \sinh w + d(A_0^3 + \]
\[ 2A_1 A_2^2 + 3A_0 A_2^3) = 0. \]

Setting the coefficients of \( \sinh^i \cosh^j w \) (\( i = 0, 1, j = 0, 1, 2, 3 \)) to zero, we get
\[ A_0 b + A_0^2 c + A_2^3 c + A_1 A_0 A_2^2 d + 3A_0 A_2^2 d + 3A_1 A_2^2 = 0, \]  
(5a)
\[ A_2 b + 2A_0 A_2 c + 3A_0^2 A_2 d + A_2^3 d = 0, \]  
(5b)
\[ A_1 b + 2A_0 A_1 c + 3A_0^2 A_1 d + 3A_1 A_0 A_2^2 d + A_1(a + k^2) \lambda^2 = 0, \]  
(5c)
\[ 2A_1 A_2 c + 6A_0 A_1 A_2 d = 0, \]  
(5d)
\[ A_1^2 c + A_2^2 c + 3A_0 A_1^2 d + 3A_0 A_2^2 d = 0, \]  
(5e)
\[ 3A_1^2 A_2 d + A_1^2 d + 2(a + k^2)A_2 \lambda^2 = 0, \]  
(5f)
\[ A_1^2 d + 3A_1 A_2^3 d + 2A_1(a + k^2) \lambda^2 = 0. \]  
(5g)

By using WU algebraic elimination method to solve the system of the overdetermined equations
(5a) - (5g) with respect to \( A_1, A_2, A_0, \lambda, k \), yields