REGIONAL BOUNDARY CONTROLLABILITY
OF HYPERBOLIC SYSTEMS.
NUMERICAL APPROACH

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ABSTRACT. The problem of regional boundary controllability for hyperbolic systems is considered. Thus, we show how one can reach a desired state given only on a part of the boundary of the system domain. Also we explore a numerical approach that leads to an explicit formula of the optimal control. The results obtained are successfully tested through computer simulations.

1. Introduction

There has been a great deal of excitement over the last few years concerning the emergence of new mathematical techniques for the regional analysis of distributed parameter system theory. The term "regional analysis" has been used to refer to control problems in which the target of our interest is not fully specified as a state, but refers only to a smaller region $\omega$ of the system domain $\Omega$. This concept has been introduced by El Jai et al. (1995) for parabolic systems and by E. Zerrik and R. Larhrissi (2000) for hyperbolic systems. Many results were obtained; in particular, it was shown that the minimum time and the transfer cost of regional controllability are less than those of the controllability on the whole domain $\Omega$. These results have been extended by E. Zerrik et al. to the case where $\omega$ is a part of the boundary of the domain of a parabolic system and consist in finding a control which steers a system (S), at time $T$, to a prescribed function defined on a subregion $\Gamma$ of the boundary $\partial \Omega$ (see [9]). Here we consider the problem of regional boundary controllability for hyperbolic systems. But it is clear that the required control depends on the time $T$ because of the finite speed of propagation, also it depends on the actuator location, the state space of (S), and the subregion $\Gamma$. Problems of this kind are encountered in various applications where the control must achieve a certain target on the boundary. This would be, for example, of particular interest in problems of elastic membrane, where the idea would be to stabilize a part of its boundary.

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The present paper gives an original treatment of the regional boundary controllability problem of hyperbolic systems and it is organized as follows. In Sec. 2 we give definitions, properties of this concept, and a link between the internal and boundary regional controllability. The next section considers the problem of boundary control to a target and is devoted to finding a control which ensures the solution of regional boundary target problem via the internal technique. In Sec. 4 we give a numerical approach which leads to an explicit formula of the optimal control. In the last section, the developed approach is illustrated by numerical simulations through an example of a two-dimensional system.

2. REGIONAL BOUNDARY CONTROLLABILITY

Let \( \Omega \) be an open bounded domain in \( \mathbb{R}^n \) \((n = 1, 2, 3)\) with a regular boundary \( \partial \Omega \). For \( T > 0 \) we denote \( Q = \Omega \times [0, T] \) and \( \Sigma = \partial \Omega \times [0, T] \) and consider the system described by the equation

\[
\begin{cases}
\frac{\partial^2 y}{\partial t^2}(x, t) + Ay(x, t) = Bu(t), & Q \\
y(x, 0) = y_0(x), & \Omega \\
\frac{\partial y}{\partial t}(x, 0) = y_1(x), & \Omega \\
\frac{\partial y}{\partial \nu_A}(\xi, t) = 0, & \Sigma
\end{cases}
\]

(1)

where \( A \) is a second-order elliptic linear symmetric operator given by

\[
A = -\sum_{i,j=1}^{n} \frac{\partial}{\partial x_i} \left( a_{ij} \frac{\partial}{\partial x_j} \right) \quad \text{with} \quad a_{ij}(x) = a_{ji}(x) \in C^1(\Omega),
\]

and there exists \( \alpha > 0 \)

such that

\[
\sum_{i,j=1}^{n} a_{ij} \xi_i \xi_j \geq \alpha \sum_{j=1}^{n} |\xi_j|^2 \quad \forall \xi = (\xi_1, \xi_2, \ldots, \xi_n) \in \mathbb{R}^n,
\]

\( \frac{\partial y}{\partial \nu_A}(\xi, t) \) denotes the conormal with respect to \( A \) and \( B \in \mathcal{L}(L^2(0, T, \mathbb{R}^p), L^2(0, T, H^1(\Omega))) \). We denote the solution of Eq. (1) by \( (y_u, \frac{\partial y_u}{\partial t}) \) and the state space \( \mathcal{F} = H^2(\Omega) \times H^1(\Omega) \). The space of controls \( U \) is chosen to be the completion of the space \( \tilde{U} = \left\{ u \in L^2(0, T, \mathbb{R}^p) \right\} \) such that \( (y_u(T), \frac{\partial y_u}{\partial t}(T)) \in \mathcal{F} \) with respect to the standard norm of \( L^2(0, T, \mathbb{R}^p) \) and \( (y_0, y_1) \in \mathcal{F} \).