LAMINAR HEAT TRANSFER IN DIFFUSER FLOW IN A COAXIAL CONICAL CHANNEL IN THE CASE OF BOUNDARY CONDITIONS OF THE FIRST KIND

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Consideration is given to the problem of convective heat transfer in slow diffuser flows in coaxial annular conical channels of constant width. A solution for thermal boundary conditions of the first kind is obtained by the method of separation of variables. The dependence of the temperature on the coordinates is represented in the form of a sum of two infinite series in confluent hypergeometric functions of the transverse coordinate that are multiplied by an exponential dependence on the longitudinal coordinate. The solution is of interest due to its being a superposition of two solutions, each having its own eigenfunctions and eigenvalues. Relations for evaluation of the initial thermal portion in the considered flows are also given.

To select the optimum design and technological parameters of die heads, it is necessary to know the special features of the flow and heat transfer of the melt in the flow elements of the molding equipment. In the extrusion method of production of strands, granules, tubes, films, etc., on the distribution section of the molding equipment the polymer melt flows in a coaxial conical channel formed by the cone of the head and the mandrel [1, 2], where the melt can cool down or warm up. Modern technologies make it possible to maintain different regimes of heat transfer at the boundaries of the channels, but experimental selection of the optimum characteristics of the process requires appreciable means. The construction of numerical models of treatment processes is not always justified either, since in many cases it is possible to obtain adequate relations between the parameters of the processes using analytical solutions. They can serve as test problems in adjusting numerical codes.

In [3, 4], the problem of isothermal flow in coaxial conical channels with different locations of the boundary surfaces is solved, while in [5, 6] a model of the flow and heat transfer in conical gaps with boundary conditions of the third kind is constructed. In this work, we investigate heat transfer in coaxial conical channels with boundary conditions of the first kind for polymer melts that behave like Newtonian fluids [7] within the ranges of the treatment parameters. In [5], it is shown that for flow rates of the liquid or dimensions of the channel of practical interest [3, 4] the Reynolds number is $Re \ll 1$, the Ném–Griffith number is $Gn \ll 1$, and the Péclet number is $Pe > 100$. These evaluations make it possible to consider the flow of the melt as a creeping flow [8] and not to take into account in the heat-transfer equation the dissipation heat and to disregard in it the change in the conductive heat flux along the stream as compared to the change in the convective heat flux and ultimately to write in a biconical system of coordinates (Fig. 1) determined by the transformation [9]

\[
\begin{align*}
  z' &= R \cos \alpha + X \sin \alpha, \\
  y' &= (R \sin \alpha - X \cos \alpha) \sin \varphi,
\end{align*}
\]
the system of equations of axisymmetric convective heat transfer in the form

\[
\frac{\partial \Pi}{\partial \xi} = \frac{1}{\sigma \partial \chi} \left( \sigma \frac{\partial v}{\partial \chi} \right),
\]

(4)

\[
\frac{\partial \Pi}{\partial \chi} = \cos(\alpha) \sin(\alpha) \frac{\sigma^2}{\gamma} v,
\]

(5)

\[
\frac{\partial}{\partial \xi} (\sigma v) = 0,
\]

(6)

\[
\text{Pe}_0 v \frac{\partial \Theta}{\partial \xi} = \frac{1}{\sigma \partial \chi} \left( \sigma \frac{\partial \Theta}{\partial \chi} \right),
\]

(7)

where \( \xi = R/h \), \( \chi = X/h \), \( v = V/V_0 \), \( V_0 = Q/(\pi h(2R_0 \sin \alpha - h \cos \alpha)) \), \( \Pi = (P - P_0)h/\gamma V_0 \), \( \sigma = \xi \sin \alpha - \chi \cos \alpha \), \( \text{Pe}_0 = V_0 h/a \), \( \Theta = (T - T_0)/(T_1 - T_0) \), and \( T_1 \) is the temperature of the channel surface formed by the external cone.

The boundary conditions are written in the form

\[
v = 0, \ \chi = 0, \ \xi_0 < \xi \leq \xi_1; \quad (8)
\]

\[
v = 0, \ \chi = 1, \ \xi_0 < \xi \leq \xi_1; \quad (9)
\]

\[
\Pi = 0, \ 0 \leq \chi \leq 1, \ \xi = \xi_0; \quad (10)
\]

\[
\Theta = 0, \ 0 \leq \chi \leq 1, \ \xi = \xi_0; \quad (11)
\]