NOTES ON AN INTERNAL BOUNDARY-LAYER HEIGHT FORMULA

Research Note

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Abstract. The derivation of the Panofsky–Dutton internal boundary-layer (IBL) height formula has been revisited. We propose that the upwind roughness length (rather than downwind) should be used in the formula and that a turbulent vertical velocity (σw) rather than the surface friction velocity (u∗) should be considered as the appropriate scaling for the rate of propagation of disturbances into the turbulent flow. A published set of wind-tunnel and atmospheric data for neutral stratification has been used to investigate the influence of the magnitude of roughness change on the IBL height.

Keywords: Diffusion analogue, Internal boundary layer, Roughness change.

Abbreviations: IBL – internal boundary layer; TKE – turbulent kinetic energy.

1. Introduction

When turbulent boundary-layer flow runs across a discontinuity in surface roughness it is no longer in equilibrium (we assume that, before the step change, it experienced homogeneity long enough to achieve equilibrium as a constant stress layer). The region of the atmosphere affected by the step change is called the internal boundary layer (IBL), of depth δ(x) in a simple two-dimensional (2D) flow. Several formulae have been in use to predict δ(x) in neutrally stratified flow, but that proposed by Panofsky (1973), and in a slightly modified form by Panofsky and Dutton (1984), is now probably the most widely accepted (see Garratt, 1992; Kaimal and Finnigan, 1994). Walmsley (1989) evaluated the accuracy of different formulae by comparison with wind-tunnel and atmospheric data sets, and showed that the Panofsky–Dutton formula gave better agreement than Elliott’s (1958) power-law formula (essentially δ ∝ x^{0.8}) and Jackson’s (1976) variant of the Panofsky formula.

2. Panofsky’s IBL Height Formulae

The Panofsky, and Panofsky–Dutton, formulae are based on the assumption of proportionality between the friction velocity and the rate of upward propagation of disturbances or adjustments of the vertical profiles of the mean flow parameters to the turbulent flow.
We can obtain an equation for the interface between incoming flow and the disturbed region (the internal boundary layer) by an argument as follows. The idea is usually attributed to Miyake (1965, unpublished thesis) and sometimes called a ‘diffusion analogue’ (Panofsky, 1973). Brutsaert (1982) points out that Monin (1959) used the same idea in connection with smoke plume spreading.

Air close to the surface first experiences the impact of roughness change; the signal then propagates upwards. Following Brutsaert (1982) we assume that the vertical rate of propagation is proportional to the variance of the vertical turbulent velocity component, $\sigma_w$ at the interface ($z = \delta(x)$), i.e.,

$$\frac{d\delta}{dt} = A\sigma_w$$  \hspace{1cm} (1)

where $A$ is a constant. Assuming that $\sigma_w \propto u_*$ Panofsky (1973) recasts the equation as

$$U(\delta)\frac{d\delta}{dx} = A\sigma_w = Bu_*$$  \hspace{1cm} (2)

where $U(\delta)$ is the mean flow speed at the height of interface, and, for steady state flows, $\delta$ is a function of downwind distance, $x$, only. For $u_*$ the Panofsky–Dutton model uses the local (in $x$) surface value $u_{*D}(x)$, where the subscript $D$ implies a value downstream of the roughness change; subscript $U$ will be used for upstream values. We can assume that the velocity at height $z = \delta$ is that given by the upstream profile, which, in the Panofsky–Dutton model, is also matched to a logarithmic profile in the IBL so that,

$$U(\delta) = \frac{(u_{*U}/\kappa)}{\ln(\delta/z_{0U})} = \frac{(u_{*D}/\kappa)}{\ln(\delta/z_{0D})}$$  \hspace{1cm} (3)

where $\kappa = 0.4$ is the von Karman constant. Streamline displacement effects due to deceleration or acceleration within the IBL are assumed small and are neglected. We could, however, include the effect of a displacement height, $d$, and replace $\delta$ by $(\delta - d)$, or if we wished to have $U = 0$ at $z = 0$, we could replace $\delta$ by $(\delta + z_0)$.

With the assumption that $B = 1.25$ ($A = 1$ and $\sigma_w = 1.25u_*$), and using the downstream roughness length and friction velocity for scaling purposes, Panofsky and Dutton’s integration of (2) gives,

$$1.25\kappa(x/z_{0D}) = (\delta/z_{0D})[\ln(\delta/z_{0D}) - 1] + 1$$  \hspace{1cm} (4)

where the initial condition that $\delta = z_{0D}$ at $x = 0$ has been applied. Panofsky’s (1973) result is similar except that $z_{0D}$ is replaced by the roughness length of the rougher of the two surfaces, the coefficient $1.25\kappa (= 0.5)$ is set equal to 0.6 and the final ‘+ 1’ term is neglected.