Study is made of the critical phenomena occurring in cracking of the interface between two different materials with initial stresses. The basic relations and complex potentials for a plane problem of the three-dimensional linearized dynamic theory of elasticity are used. The exact solution is obtained for the case of equal roots (complex parameters). The basic mechanical effects are analyzed.

Introduction. The paper [9] formulates problems on critical phenomena occurring in cracking (at a constant rate) of the interface between two different materials with initial stresses, analyzes results obtained by more simple models (a homogeneous material in the classical linear theory of elasticity and a homogeneous material in the linearized theory of elasticity for materials with initial stresses), and proposes a method for solving the posed problems under plane-strain conditions. The approach from [9] employs the basic relations of a dynamic plane problem of the three-dimensional linearized theory of elasticity, e.g., [1–3, 8], in a unified form common for compressible and incompressible elastic bodies with an arbitrary elastic potential and represents the stresses and displacements of a dynamic plane problem of the three-dimensional linearized theory of elasticity in terms of complex potentials. In contrast to [1–3, 8] where those stress and displacement representations for dynamic problems were expressed in terms of two analytical functions of complex variables, the paper [9] derived and used complex stresses and displacements for a half-plane in terms of one analytical function defined on the entire plane. Based on [9], the exact solution to the problem posed was obtained in [10] for the case of unequal complex roots (as applied to each of the materials whose interface cracks) in a unified form common for the theory of finite (large) initial strains and two versions of the theory of small initial strains. Based on the exact solution, the basic mechanical effects were also analyzed in [10]. From the analysis, it follows that the critical phenomena occur when the cracking rate tends to the velocity of the Rayleigh wave in a less rigid material (of the two being considered) with initial stresses.

The objective of the present paper is to construct the exact solution to the problem in the case of equal complex roots (as applied to each of the materials whose interface cracks), which will also be represented in a unified form common for the theory of finite (large) initial strains and two versions of the theory of small initial strains. Again, the approach from [9] will be used and the basic mechanical effects will be analyzed.

1. Basic Relations. Following [9, 10], let us briefly recall the formulation of problems, focusing on the relations for the case of equal complex roots (complex parameters). Thus, consider two half-planes \( y_2 > 0 \) and \( y_2 < 0 \) of the plane \( y_1 O y_2 \) (Fig. 1) connected along the boundary line (plane) \( y_2 = 0 \). The half-planes are made of different materials with initial stresses. The superscripts “(±)” will label all quantities pertaining to the upper, \( D^{(+)}, \) and lower, \( D^{(-)} \), half-planes, and the superscript “0” will label all quantities pertaining to the initial stress–strain state. Let compressible elastic bodies with an arbitrary elastic potential be isotropic or orthotropic. For orthotropic bodies, we additionally assume that the axes of symmetry of the material properties coincide with the axes of the chosen coordinate system. We will also adopt all the assumptions made in [9] in formulating the
problem, in particular, that there are no initial stresses in the planes $y_2 = \text{const}$ (planes parallel to the interface); if the initial state is homogeneous, then the assumptions have the form

$$\sigma_{22}^{0(\pm)} = 0, \quad \sigma_{11}^{0(\pm)} = \text{const}, \quad \sigma_{33}^{0(\pm)} = \text{const}. \quad (1.1)$$

As in [9, 10], let the interface $y_2 = 0$ contain a crack that is infinite along the $Oy_3$-axis (perpendicular to the picture plane) and semi-infinite along the $Oy_1$-axis ($y_1 \leq 0, y_2 = 0, -\infty < y_3 < +\infty$). The crack moves along the $Oy_1$-axis (in the interfaces) from left to right at a constant rate $v$. Within the framework of the mechanics of composites, the problem in question corresponds to the problem on the local effects near the tip of a microcrack (its dimensions are much smaller than the layer thicknesses) moving in the interface of a composite.

Let us introduce a mobile Cartesian coordinate system with coordinates $\eta_j$ ($j = 1, 2, 3$) and the origin coinciding with the crack tip. Assume that the mobile coordinate system moves together with the crack tip from left to right at a constant rate $v$. Thus, we have the relations

$$\eta_1 = y_1 - vt, \quad \eta_2 = y_2, \quad \eta_3 = y_3. \quad (1.2)$$

In accordance with the problem formulation [9, 10] and Fig. 1, for a semi-infinite crack ($\eta_1 \leq 0, \eta_2 = 0, -\infty < \eta_3 < +\infty$) in the mobile coordinate system, the boundary conditions in the plane $\eta_1 \eta_2$ can be written for $\eta_2 = 0$ as

$$Q_{22}^{(\pm)} = 0, \quad Q_{21}^{(\pm)} = 0 \quad \text{for} \quad \eta_2 = 0 \quad \text{and} \quad \eta_1 \leq 0,$$

$$Q_{22}^{(\pm)} = Q_{22}^{(-)}, \quad Q_{21}^{(\pm)} = Q_{21}^{(-)}, \quad \frac{\partial}{\partial \eta_1} (u_1^{(\pm)} - u_1^{(-)}) = 0,$$

$$\frac{\partial}{\partial \eta_1} (u_2^{(\pm)} - u_2^{(-)}) = 0 \quad \text{for} \quad \eta_2 = 0 \quad \text{and} \quad \eta_1 > 0. \quad (1.3)$$

The boundary conditions (1.3) on the crack surface (for $\eta_2 = 0$ and $\eta_1 \leq 0$) correspond to an unloaded crack and are homogeneous. As in [1–4, 8–10], in (1.3) and below, the prime near stresses, $Q_{ij}'$, implies that these stresses are referred to the area elements in the initial stress–strain state.

Hereafter, following [1–4, 8–10], we will use complex variables that in a plane dynamic problem of the linearized theory of elasticity for prestressed materials are introduced as follows:

$$z_j = \eta_1 + i\mu_j \eta_2 = y_1 - vt + i\mu_j y_2, \quad j = 1, 2,$$

$$\bar{z}_j = \eta_1 + \bar{i}\mu_j \eta_2 = y_1 - vt + \bar{i}\mu_j y_2, \quad (1.4)$$