LETTER

Perturbative Evaluation of the Effective Action for a Self-Interacting Conformal Field on a Manifold with Boundary

George Tsoupros

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In a series of three projects a new technique which allows for higher-loop renormalisation on a manifold with boundary has been developed and used in order to assess the effects of the boundary on the dynamical behaviour of the theory. Commencing with a conceptual approach to the theoretical underpinnings of the underlying, spherical formulation of Euclidean Quantum Field Theory this overview presents an outline of the stated technique’s conceptual development, mathematical formalism and physical significance.

KEY WORDS: Quantum field theory; manifold with boundary; effective action.

The investigation of the effects generated on the dynamical behaviour of quantised matter fields by the presence of a boundary in the background geometry is an issue of central importance in Euclidean Quantum Gravity. This issue arises naturally in the context of any evaluation of radiative corrections to a semi-classical tunnelling geometry and has been studied at one-loop level through use of heat

2School of Physics, The University of New South Wales, New South Wales 2052, Australia.
3Permanent address: gts@gscas.ac.cn, Department of Physics, The Graduate School of the Chinese Academy of Sciences, Yuquan Road, No. 19A, P.O. Box 3908, Beijing 100039, People’s Republic of China. E-mail address: gts@phys.unsw.edu.au
kernel and functional techniques. These methods were subsequently extended in the presence of matter couplings. Despite their success, however, such techniques have limited significance past one-loop order. Not only are explicit calculations of higher-order radiative effects far more reliable for the qualitative assessment of the theory’s dynamical behaviour under conformal rescalings of the metric but they are, in addition, explicitly indicative of boundary related effects on that behaviour. Such higher-order calculations necessarily rely on diagrammatic techniques on a manifold with boundary. Fundamental in such a calculational context is the evaluation of the contribution which the boundary of the manifold has to the relativistic propagator of the relevant quantised matter field coupled to the manifold’s semi-classical background geometry. It would be instructive, in this respect, to initiate an approach to such a higher loop-order renormalisation on a manifold with boundary by outlining the considerations which eventuated in the “background field” method, that approach to metric quantisation which is predicated on a fixed geometrical background.

The analysis relevant to the background field method can most easily be exemplified in the case of a massless scalar field minimally coupled to the background geometry. In the case of flat Euclidean space—defined by the analytical extension which eventuates in the replacement of $x_0$ by $-ix_0$—the generating functional relevant to the massless scalar field $\phi$ coupled to a classical source $J$ is

$$Z[J] = \int D[\phi] e^{-\int d^4x [\frac{1}{2}\phi \Box \phi - J\phi]}$$  \hspace{1cm} (1)

which, upon Gaussian integration yields

$$Z[J] = [\det(\partial^2)]^{-\frac{1}{2}} e^{\int d^4xd^4y [J(x)\Delta(x,y)J(y)]}$$  \hspace{1cm} (2)

as a result of which, the scalar propagator of momentum $k$

$$\Delta(x, y) = \int \frac{d^4k}{(2\pi)^4} \frac{e^{i(k(x-y))}}{k^2}$$  \hspace{1cm} (3)

The generating functional in the presence of gravity with a minimal coupling between the massless scalar field and the background metric $g_{\mu\nu}$, which for the sake of mathematical consistency in the context of the present formalism is taken to have a Euclidean signature, is

$$Z[J] = \int D[\phi] e^{-\int d^4x [\frac{1}{2}\phi \Box \phi - J\phi \sqrt{g} - \frac{1}{2} \sqrt{g} R \phi + \frac{1}{2} J \phi \sqrt{g}]}$$  \hspace{1cm} (4)

where the Riemannian manifold has been assumed, without loss of generality, to have no boundary so as to allow for vanishing surface terms. Again, Gaussian