Effective Gravity in Randall–Sundrum Infinite Brane World

T. Tanaka

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The gravity induced on the brane in the Randall–Sundrum (RS) infinite brane world is briefly reviewed. We also discuss the possibility of the absence of black hole configuration in this model based on the argument of the AdS/CFT correspondence.

KEY WORDS: brane-world; black hole; AdS/CFT correspondence.

1. INTRODUCTION

Current candidates for the fundamental theory of particle physics, such as string theory or M-theory, are all defined as a theory in higher dimension. To obtain an appropriate 4D effective theory starting with such a theory, a certain dimensional reduction is necessary. One well-known scheme of dimensional reduction is the Kaluza–Klein compactification. In this scheme, the size of the extra dimension is supposed to be very small so as not to excite the modes which have momentum in the direction of the extra dimension. This scheme seems to work well as a mechanism to shield the effect of extra dimensions. This Kaluza–Klein scheme, however, is not a unique possible scheme for dimensional reduction. Recently, the brane-world scenario has been attracting a lot of attention as an alternative possibility (Antoniadis et al., 1998; Arkani-Hamed et al., 1998; Horana and Witten, 1996a,b; Randall and Sundrum, 1999a,b). The essential feature of the brane world distinct from the ordinary Kaluza–Klein compactification is that the matter fields of the standard model are supposed to be localized on the brane, while the graviton can propagate in a higher dimensional space–time which we call “bulk.” Owing to the assumption that the ordinary matter fields are localized on the brane, the brane-world models can be consistent with the particle physics experiments even if the length scale of the extra dimension is not extremely small. Then, the gravity is possibly altered at a rather macroscopic length scale, while the observational constraint about the deviation from the 4D general relativity obtained so far is

1 Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan.
not severely below sub-mm scale. Hence, a characteristic length scale can be as large as sub-mm scale in the brane world scenario. Therefore it may open up the possibility of observing the evidence of the existence of an extra dimension.

In the course of studies on brane-world, a new scenario was proposed by Randall and Sundrum (RS, 1999a,b). One of the novel ideas of their new proposal is that the gravity can be effectively localized as a result of the warped compactification, even though the extension of the extra dimension is infinite. Although the recovery of the 4D general relativity in this model is not so automatic, so far any results which are significantly distinguishable from the standard ones have not been reported. In this paper, we review the current status of the studies on the gravity in this model.

This paper is organized as follows. In Section 2 we explain the setup of the RS model with infinite extra dimension. In Section 3 we review the geometrical approach to the gravity in this model, finding the limitation of the approach in which we do not solve the 5D equations of motion. In Sections 4 and 5 we summarize the results for linear perturbations and for nonlinear perturbations of this model, respectively. In Section 6 we discuss the possibility that there is no static black hole solution in this model, applying the argument of the AdS/CFT correspondence. In Section 7 we give a brief summary.

2. WARPED EXTRA DIMENSION

In this section, we explain the model proposed by Randall and Sundrum (1999b). In this model, 5D Einstein gravity with negative cosmological constant $\Lambda$ is assumed. The ordinary matter fields are confined on a 4D object called "brane." This brane has positive tension $\sigma$, and the space–time has reflection symmetry ($Z_2$-symmetry) at the position of this brane $y = y_b$. Here $y$ is the Gaussian normal coordinate in the direction perpendicular to the brane. The 5D Einstein equations are

$$^{(5)}G_{ab} = -\Lambda g_{ab} + 8\pi G_5 S_{ab} \delta(y - y_b), \quad (1)$$

with

$$S_{ab} = -\sigma \gamma_{ab} + T_{ab}, \quad (2)$$

where $T_{ab}$ is the energy–momentum tensor of the matter field localized on the brane, $\gamma_{ab}$ is the 4D metric induced on the brane, and $G_5$ is the 5D Newton’s constant.

One solution of (1) is 5D anti-de Sitter (AdS) space

$$ds^2 = dy^2 + e^{-2|y|/\ell} (-dt^2 + dx^2), \quad (3)$$

with a single positive tension brane located at $y = 0$. Here, $\ell$ is the curvature radius of 5D AdS space. The 5D cosmological constant and the brane tension are