The bicovariant differential calculus on four-dimensional \( \kappa \)-Poincare group and corresponding Lie-algebra like structure for any metric tensor are described. The bicovariant differential calculus on four-dimensional \( \kappa \)-Weyl group and corresponding Lie-algebra like structure for any metric tensor in the reference frame in which \( g_{00} = 0 \) are considered.

1 Introduction

One of the important problems is the construction of the bicovariant differential calculus on \( \kappa \)-Poincaré group. Using an elegant approach due to Woronowicz [1], the differential calculi on four-dimensional Poincaré group for diagonal metric tensor [2] and three-dimensional [3], as well as on the Minkowski space [2] and [4] were constructed.

In the paper [5] the \( \kappa \)-deformation of the Poincaré algebra and group for arbitrary metric tensor has been described and under the assumption that \( g_{00} = 0 \) the \( \kappa \)-deformation of the Weyl group as well as algebra has been constructed.

In this paper in section 2 we briefly sketch the construction of differential calculus on the \( \kappa \)-Poincaré group for any metric tensor \( g_{\mu \nu}, \mu, \nu = 0, \ldots, 3 \). We obtain the corresponding Lie algebra structure and prove its equivalence to the \( \kappa \)-Poincaré algebra. In section 3 we present the construction of differential calculus on the \( \kappa \)-Weyl group for any metric tensor \( g_{\mu \nu}, \mu, \nu = 0, \ldots, 3 \) with \( g_{00} = 0 \). We find the corresponding Lie algebra structure and prove its equivalence to the \( \kappa \)-Weyl algebra.

2 The \( \kappa \)-Poincaré group and algebra

We assume that the metric tensor \( g_{\mu \nu}, (\mu \nu = 0, 1, \ldots, 3) \) is represented by an arbitrary nondegenerate symmetric \( 4 \times 4 \) matrix (not necessary diagonal) with \( \det(g_{\mu \nu}) = \pm 1 \) (in more general case in some equations we use the parameter \( \det(g) \)).

The Poincaré group \( \mathcal{P} \) consists of the pairs \( (x, \Lambda) \), where \( x \) is a 4-vector, \( \Lambda \) is the matrix of the Lorentz group in 4-dimensions, with the composition law:

\[
(x^\mu, A^\mu_\nu) \ast (x'^\nu, \Lambda'^\nu_\sigma) = (A'^\mu_\nu x'^\nu + x^\mu, A'^\mu_\nu A^\nu_\sigma).
\]

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Karol Przanowski

The κ-Poincaré group is a Hopf *-algebra defined as follows [5]. Consider the universal *-algebra with unity, generated by selfadjoint elements $\Lambda^\mu, x^\mu$ subject to the following relations:

\[
[A^\alpha_\rho, x^\xi] = -\frac{i}{\kappa}((\Lambda^\alpha_\rho - \delta^\alpha_\rho)A^\xi_\rho + (\Lambda^\xi_\rho - g^\xi_\rho)\delta^\alpha_\rho),
\]

\[
[x^\xi, A^\alpha_\rho] = \frac{i}{\kappa}(\delta^\xi_\rho x^\alpha - \delta^\alpha_\rho x^\xi), \quad [A^\alpha_\rho, \Lambda^\mu_\nu] = 0.
\]

The comultiplication, antipode and counit are defined as follows:

\[
\Delta \Lambda^\mu_\nu = \Lambda^\mu_\alpha \otimes \Lambda^\nu_\alpha, \quad \Delta x^\mu = x^\mu \otimes x^\nu + x^\mu \otimes I,
\]

\[
S(\Lambda^\mu_\nu) = \Lambda^\mu_\nu, \quad S(x^\mu) = -\Lambda^\mu_\nu x^\nu,
\]

\[
\epsilon(\Lambda^\mu_\nu) = \delta^\mu_\nu, \quad \epsilon(x^\mu) = 0.
\]

In our construction of the bicovariant *-calculi we use the Woronowicz theory, [1]. First we construct the right ad-invariant ideal $R$ in ker $\varepsilon$, $(ad(R) = R \otimes \mathcal{P}_\kappa)$. We put:

\[
\Delta^\mu_\nu = \Lambda^\mu_\nu - \delta^\mu_\nu,
\]

\[
\Delta^\mu_\nu = x^\alpha \Delta^\mu_\alpha + \frac{i}{\kappa}(g^\mu_\nu \Delta^\alpha_\mu - \delta^\mu_\nu \Delta^\alpha_\nu),
\]

\[
x^\alpha \beta = x^\alpha x^\beta + \frac{i}{\kappa}(g^\alpha_\beta x^\alpha - \delta^\alpha_\beta x^\alpha),
\]

\[
\Delta^\mu_\nu = \Delta^\alpha_\nu - \frac{1}{8}x^\alpha x^\beta \epsilon_{\gamma\delta\varepsilon\zeta} x^{\beta\gamma\delta\varepsilon\zeta},
\]

\[
x^\alpha \beta = \frac{1}{4}x^\alpha x^\beta,
\]

\[
Theorem 1. R has the following properties:
\]

(i) $R$ is ad-invariant,

(ii) for any $a \in R$, $S(a)^* \in R$,

(iii) ker $\varepsilon/R$ is spanned by the following elements:

\[
x^\mu, \quad \Delta^\mu_\nu, \mu < \nu; \quad \varphi = x^\mu; \quad \varphi_\mu = \epsilon_{\mu\alpha\beta\gamma} \Delta^\alpha_\beta \Delta^\gamma_\mu.
\]

Note only that:

(a) $\Delta^\mu_\nu, x^\mu$ are "improved" generators of (ker $\varepsilon)^2$ which form the (not completely reducible) multiplet under adjoint action of $\mathcal{P}_\kappa$,

(b) if $g_0 \neq 0$ then the ideal generated by $\Delta^\alpha_\beta \Delta^\mu_\nu, \Delta^\mu_\nu, x^\mu$ equals ker $\varepsilon$. In order to obtain reasonable calculus (in the sense that it contains all differentials $dx^\mu, d\Lambda^\mu_\nu$) we have subtracted the trace of $x^\mu \vee$ and completely antisymmetric part of $\Delta^\mu_\nu$.  