On the Efficiency of a Global Non-differentiable Optimization Algorithm Based on the Method of Optimal Set Partitioning

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Abstract. The examined algorithm for global optimization of the multiextremal non-differentiable function is based on the following idea: the problem of determination of the global minimum point of the function $f(x)$ on the set $\Omega(f(x)$ has a finite number of local minima in this domain) is reduced to the problem of finding all local minima and their attraction spheres with a consequent choice of the global minimum point among them. This reduction is made by application of the optimal set partitioning method. The proposed algorithm is evaluated on a set of well-known one-dimensional, two-dimensional and three-dimensional test functions. Recommendations for choosing the algorithm parameters are given.

Key words: global minimum, non-differentiable optimization, optimal set partitioning

1. Introduction

Because of the variety of practical global optimization problems, it is expedient to apply specific methods of global search that most completely take into account characteristics of a particular class of problems. This fact follows from the numerous publications, for example Batishev (1975), Dem'yanov and Vasil'ev (1981), Strongin (1978), Suharev (1981, 1989), Ziglyavskiy (1985), Ziglyavskiy and Zilinskas (1991), Zilinskas (1986), Zilinskas and Shaltyanis (1989), reflecting the modern state of theory and methodology of global optimization.

Due to such variety, there is no uniform standard classification of global methods. Each of known classifications has its advantages and its drawbacks. However, in each classification that claims its own completeness, the approaches such as “multistart” Ziglyavskiy and Zilinskas (1991) or such as covering methods Suharev (1989) are discussed. These approaches are reduced to the estimation of the attraction spheres of local minima and to the choice of initial points for consecutive or parallel local descent to the local minimum points with the subsequent choice of the global minimum point among them. Let us examine these approaches explicitly.

Let us formulate the global optimization problem following Strongin (1978).
Let $f(x)$ be a real, multiextremal, continuous function defined on the domain $\Omega$ in an $n$-dimensional Euclidean space $E_n$, where the number of local minima is finite and does not exceed $N$.

Let us consider the problem of searching for the point $x^* \in \Omega$ (assumed to exist) such that

$$f(x^*) = \min_{x \in \Omega} f(x).$$  \hfill (1)

If such a point $x^*$ exists in domain $\Omega$ and in a certain vicinity $U(x^*)$

$$f(x^*) \leq f(x), \quad x \in \Omega \cap U(x^*),$$

then function $f(x)$ is called unimodal in $U(x^*)$.

If there are several points $\tau_i^*, \ 1 \leq i \leq N$, in the domain of definition $\Omega$ and each of them has its own vicinity $U(\tau_i^*)$ such that

$$f(\tau_i^*) \leq f(x), \quad x \in \Omega \cap U(\tau_i^*),$$  \hfill (2)

then function $f(x)$ is called multiextremal.

The points from (2) are called local minimum points, and the point $\tau^*$ is a global minimum.

Let $f(x)$ be the unimodal function on a certain subset $\Omega_i, \ 1 \leq i \leq N$, of the domain $\Omega$ (the local minimum point of the function $f(x)$ on the subset $\Omega_i$ is denoted by $\tau_i^*$), and let

$$\bigcup_{i=1}^N \Omega_i = \Omega.$$  

Then, by applying the known local descent methods, for an arbitrary initial point $\tau_i^0 \in \Omega_i$, we can obtain the corresponding local minimum point $\tau_i^* \in \Omega_i$. It is said that the subset $\Omega_i$ is an attraction sphere of the local minimum $\tau_i^*$. In other words, the local minimum attraction sphere is a sphere in which the steepest descent starting from any point of this sphere leads to this local minimum.

Thus, the global optimization problem (1) is solved once the domain of definition $\Omega$ is partitioned into attraction spheres $\Omega_i$ of the local minima $\tau_i^*, \ 1 \leq i \leq N$.

However, as many authors indicate, for example Batishev (1975), Strongin (1978), Ziglyavskiy and Zilinskas (1991), Zilinskas (1986), the evaluation for the partitioning the domain of definition $\Omega$ into attraction spheres $\Omega_i, \ 1 \leq i \leq N$ is a complex problem.

Besides, in the general case, obtaining the evaluation for the number $N$ of attraction spheres is a hard problem. Thus, the development of the objective function models and methods that include the number of local minima with characteristics of their attraction spheres is still an issue, Ziglyavskiy and Zilinskas (1991).