OSCILLATIONS OF HIGHER ORDER DIFFERENTIAL EQUATIONS OF NEUTRAL TYPE

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Abstract. In this paper, sufficient conditions have been obtained for oscillation of solutions of a class of nth order linear neutral delay-differential equations. Some of these results have been used to study oscillatory behaviour of solutions of a class of boundary value problems for neutral hyperbolic partial differential equations.

Keywords: Oscillation, nonoscillation, boundary value problem, neutral equations, hyperbolic equations.

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1. During the last few years many authors have obtained sufficient conditions for oscillation of solutions of neutral differential equations of higher orders (see [1, 2, 6, 8]). The conditions assumed differ from authors to authors due to the different techniques they use and different type of equations they consider. It is interesting to note that the conditions assumed by different authors for a similar type of equations are often not comparable. In a recent paper [6], P. K. Mohanty and the author have considered oscillation of solutions of a class of linear homogeneous neutral differential equations of order $n$. In the present work we consider equations of the form

\begin{equation}
(y(t) - py(t - \tau))^n + \sum_{i=1}^{m} q_i(t)y(t - \tau_i(t)) = 0,
\end{equation}

where $0 < p < 1$, $\tau > 0$ and $\tau_i$, $q_i \in C([0, \infty), \mathbb{R})$, $1 \leq i \leq m$, such that $\tau_i(t) \geq 0$. These equations and the conditions assumed here are different from those in earlier works.

By a solution of (1) we mean a real-valued continuous function $y$ on $[T_y, \infty)$ for some $T_y > 0$ such that $(y(t) - py(t - \tau))$ is $n$-times continuously differentiable.
and (1) is satisfied for $t \in [T_0, \infty)$. Such a solution is said to be oscillatory if it has arbitrarily large zeros; otherwise, it is called nonoscillatory. Eq. (1) is oscillatory if all its solutions are oscillatory.

In Section 2 sufficient conditions are obtained for oscillation of solutions of (1). Some of the results of this section are used to predict oscillation of some Neumann and Dirichlet boundary value problems for neutral hyperbolic partial differential equations in Section 3.

We need the following lemmas for our work:

**Lemma 1.1.** [7] If

$$(H_1) \quad 0 \leq q_i(t) \leq q_0, \quad 0 \leq \tau_i(t) \leq \tau_0, \quad t \in [0, \infty), \quad 1 \leq i \leq m,$$

where $q_0$ and $\tau_0$ are positive constants, and

$$(H_2) \quad \lim_{\lambda \to 0} \inf_{i=1}^m \left( \lambda^{-1} \sum_{i=1}^m q_i(t) \exp(\lambda \tau_i(t)) \right) > 1,$$

then (2) is oscillatory, where

$$(2) \quad x'(t) + \sum_{i=1}^m q_i(t)x(t - \tau_i(t)) = 0.$$