QUANTUM FIELDS IN THE SPACETIME TANGENT BUNDLE

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Maximal-acceleration invariant quantum fields are formulated in terms of the differential geometric structure of the spacetime tangent bundle. The simple special case is considered of a flat Minkowski spacetime for which the bundle is also flat. The field is shown to have a physically based Planck-scale effective regularization and a spectral cutoff at the Planck mass.

Key words: quantum field theory, spacetime tangent bundle, maximal proper acceleration, elementary particle spectrum, Planck mass.

1. INTRODUCTION

Some time ago, it was demonstrated [1,2] that a vanishing directional derivative of a maximal-acceleration invariant scalar field along particle trajectories in the spacetime tangent bundle [3–6] is identical in form to the covariant Vlasov equation in curved spacetime in the presence of both gravitational and nongravitational forces. It is also of interest to consider a maximal-acceleration invariant scalar field \( \phi \) defined on the spacetime tangent bundle, which is vanishing when acted on by the invariant eight-dimensional d'Alembertian operator \( (8)\Box \), defined on the spacetime tangent bundle. Thus one has the bundle wave equation:

\[
(8)\Box \phi = 0,
\]

where the eight-dimensional d'Alembertian (or Laplace-Beltrami) operator is defined by
Here $G^{MN}$ is the bundle metric tensor [3-6], and the coordinates $x^M$ of a generic point in the bundle manifold are defined by

$$\{x^M; M = 0,1,\ldots,7\}$$

$$= \{x^\mu, x^m; \mu = 0, 1, 2, 3; m = 4, 5, 6, 7\}$$

$$\equiv \{x^\mu, \rho_0 v^\mu; \mu = 0, 1, 2, 3\}$$

where $x^\mu$ and $v^\mu$ are the spacetime and four-velocity coordinates, respectively. Greek indices refer to spacetime and range from 0 to 3; lower case Latin indices refer to four-velocity space and range from 4 to 7; and upper case Latin indices refer to a point in the bundle and range from 0 to 7. Any lower case Latin index $n$ appearing in a canonical spacetime tensor or connection is defined to be $n - 4$ implicitly. The bundle metric is given by [3-6]

$$G^{MN} = \begin{bmatrix}
    g_{\mu\nu} + g_{\alpha\beta} A^\alpha_\mu A^\beta_\nu & A_\mu \\
    A_m \nu & g_{mn}
\end{bmatrix},$$

where $g_{\mu\nu}$ is the spacetime metric tensor, $A^\mu_\nu$ is the gauge potential [3-7]

$$A^\mu_\nu = \rho_0 v^\lambda \Gamma^\mu_\lambda\nu,$$

and $\Gamma^\mu_\lambda\nu$ is the spacetime affine connection. The length $\rho_0$ is of the order of the Planck length and is given explicitly by

$$\rho_0 = \frac{c^2}{a_0} = \frac{(\hbar G/c^3)^{1/2}}{2\pi\alpha},$$

where $a_0$ is the maximum possible proper acceleration relative to the vacuum, $c$ is the velocity of light in vacuum, $\hbar$ is Planck's constant divided by $2\pi$, and $G$ is the universal gravitational constant [4,5,8]. The constant $\alpha$ is a dimensionless number of order unity that according to fundamental string theory is given by [4]

$$\alpha = \left(\frac{1}{\pi b}\right) \left(\frac{G}{\hbar c a'}\right)^{1/2},$$