INHOMOGENEOUS DARK RADIATION DYNAMICS ON A DE SITTER BRANE

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Abstract. Assuming spherical symmetry we analyse the dynamics of an inhomogeneous dark radiation vacuum on a Randall and Sundrum 3-brane world. Under certain natural conditions we show that the effective Einstein equations on the brane form a closed system. On a de Sitter brane and for negative dark energy density we determine exact dynamical and inhomogeneous solutions which depend on the brane cosmological constant, on the dark radiation tidal charge and on its initial configuration. We also identify the conditions leading to the formation of a singularity or of regular bounces inside the dark radiation vacuum.

1. Introduction

In the context of the intensive search for extra dimensions the Randall and Sundrum (RS) brane world scenario is specially compelling for its simplicity and depth (Randall and Sundrum, 1999). In this scenario the observable Universe is a 3-brane boundary of a non-compact $Z_2$ symmetric 5-dimensional Anti-de Sitter (AdS) space. The matter fields are restricted to the brane but gravity exists in the whole AdS bulk and is bound to the brane by the warp of the infinite fifth dimension.

In recent years numerous studies have been conducted within the RS scenario [see (Maartens, 2001) for a recent review and notation]. From an effective 4-dimensional point of view (Shiromizu, Maeda and Sasaki, 2000; Sasaki, Shiromizu and Maeda, 2000; Maartens, 2000) the interactions existing between the brane and the bulk lead to a modification of the Einstein equations by a set of two distinct terms, namely a local high energy embedding term generated by the matter energy-momentum tensor and a non-local term induced by the bulk Weyl tensor. The resulting equations have a complex non-linear dynamics. For example the exterior vacuum of a collapsing distribution of matter on the brane is now filled with modes originated by the bulk Weyl curvature and can no longer be a static space (Bruni, Germani and Maartens, 1999; Govender and Dadhich, 2002).

Previous investigations of the RS scenario have been focused on static or homogeneous dynamical solutions. In this proceedings we analyse some aspects of the inhomogeneous dynamics of a RS brane world vacuum [see Neves and Vaz., 2002, for more details].

2. Vacuum Einstein Equations on the Brane

In the 4-dimensional effective geometric approach to the RS brane world scenario first introduced by Shiromizu, Maeda and Sasaki (Shiromizu, Maeda and Sasaki, 2000; Sasaki, Shiromizu and Maeda, 2000; Maartens, 2000), the induced Einstein vacuum field equations on the brane are given by

$$G_{\mu\nu} = -\Lambda g_{\mu\nu} - \mathcal{E}_{\mu\nu},$$  

where $\Lambda$ is the brane cosmological constant and the tensor $\mathcal{E}_{\mu\nu}$ is the limit on the brane of the projected 5-dimensional Weyl tensor. On one hand the Weyl symmetries ensure it is a symmetric and traceless tensor. On the other the Bianchi identities constrain it to satisfy the conservation equations

$$\nabla_{\mu} \mathcal{E}^{\mu}_{\nu} = 0.$$  

The tensor $\mathcal{E}_{\mu\nu}$ can be written in the following general form (Maartens, 2000)

$$\mathcal{E}_{\mu\nu} = -\left(\frac{k}{k'}\right)^4 \left[ U(u_{\mu} u_{\nu} + \frac{1}{3} h_{\mu\nu}) + P_{\mu\nu} + Q_{\mu \nu} + Q_{\nu \mu}\right],$$  

where $u_\mu$ such that $u^\mu u_\mu = -1$ is the 4-velocity field and $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ is the tensor which projects orthogonally to $u_\mu$. The forms $U$, $P_{\mu\nu}$ and $Q_{\mu}$ represent different characteristics of the effects induced on the brane by the free gravitational field in the bulk. Thus, $U$ is interpreted as an energy density, $P_{\mu\nu}$ as stress and $Q_{\mu}$ as energy flux.

Since the 5-dimensional metric is unknown, in general $\mathcal{E}_{\mu\nu}$ is not fully determined on the brane (Shiromizu, Maeda and Sasaki, 2000; Sasaki, Shiromizu and Maeda, 2000). As a consequence the effective 4-dimensional theory is not closed and to close it we need simplifying assumptions about the bulk degrees of freedom. For example we may consider a static and spherically symmetric brane vacuum with $Q_{\mu} = 0$, $P_{\mu\nu} \neq 0$ and $U \neq 0$ to find the Reissner-Nordström black hole solution on the brane (Dadhich, Maartens, Papadopoulos and Rezania, 2000). Let us now show that it is possible to take a non-static spherically symmetric brane vacuum with $Q_{\mu} = 0$, $U \neq 0$, $P_{\mu\nu} \neq 0$ and still close the system of dynamical equations.

Consider the general, spherically symmetric metric in comoving coordinates $(t, r, \theta, \phi)$,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -e^\sigma dt^2 + A^2 dr^2 + R^2 d\Omega^2,$$  

where $d\Omega^2 = d\theta^2 + \sin^2 \theta d\phi^2$, $\sigma = \sigma(t, r)$, $A = A(t, r)$, $R = R(t, r)$ and $R$ is interpreted as the physical spacetime radius. If the stress is isotropic then the traceless $P_{\mu\nu}$ will have the general form

$$P_{\mu\nu} = P \left( r_{\mu} r_{\nu} - \frac{1}{3} h_{\mu\nu} \right),$$  

where