ELASTIC WAVE SCATTERING BY AN INTERFACE CRACK BETWEEN A PIEZOELECTRIC LAYER AND AN ELASTIC SUBSTRATE

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Abstract. The scattering problem of a plane elastic wave by an interface crack between a piezoelectric layer and an elastic substrate is analyzed by means of the integral transform and the Cauchy singular integral equation methods. The effects of the crack configuration, the incident direction of the wave and the material combinations are examined.

1. Formulation of the problem. Consider a piezoelectric layer of infinite length and of $h$ thickness, perfectly bonded to a half-space elastic substrate, as shown in Fig. 1. Assume that a crack of $2c$ length lies on the interface. Refer to a Cartesian coordinate system $(x, y, z)$ located at the center of the crack, in which the $z$-axis is the poling axis of the piezoelectric medium. Assume that a wave propagates from the elastic substrate with an incidence angle $\theta$ with respect to the $z$-axis (Fig. 1).

![Diagram](image)

Figure 1. Crack configuration.

In the case of plane strain, the linear constitutive relations of a piezoelectric material transversely isotropic with respect to the $z$-axis are expressed as

\[
\begin{align*}
\sigma_{xx} &= c_{11} u_x + c_{13} w_z + e_{13} \phi_z, \\
\sigma_{yy} &= c_{11} u_y + c_{33} w_z + e_{33} \phi_z, \\
\sigma_{xz} &= c_{44} u_z + e_{44} w_x + e_{15} \phi_x, \\
D_x &= e_{13} u_x + e_{15} w_z - \kappa_{11} \phi_x, \\
D_z &= e_{13} u_z + e_{33} w_x - \kappa_{33} \phi_z.
\end{align*}
\]

(1)
where $\phi$ denotes the electric potential, $u$ and $w$ denote the displacements in the $x$ and $z$ directions, respectively, $\sigma_{xx}$, $\sigma_{zz}$, and $\sigma_{xz}$ the stresses, $D_x$ and $D_z$ the electric displacements, $c_{11}$, $c_{13}$, $c_{33}$ and $c_{44}$ the elastic moduli, $e_{13}$, $e_{15}$ and $e_{16}$ the piezoelectric coefficients, and $\kappa_{11}$ and $\kappa_{33}$ the dielectric coefficients. The constitutive relations of the isotropic medium are written as

$$\sigma_{xx}^e = (\lambda + 2G)u_{xx} + \lambda w_{xz}^e, \quad \sigma_{zz}^e = \lambda u_{zx} + (\lambda + 2G)w_{zz}^e, \quad \sigma_{xz}^e = Gw_{x} + Gw_{xz}^e, \quad (2)$$

where the superscript $e$ stands for quantities of the elastic substrate, $\lambda$ the Lamé constant, and $G$ the shear modulus.

The governing equations of the piezoelectric material read (Qin, 2000)

$$c_{11}u_{xx} + c_{44}u_{xz} + (c_{13} + c_{44})w_{xx} + (e_{13} + e_{14})\phi_{xz} = \rho \frac{\partial^2 u}{\partial t^2},$$

$$(c_{13} + c_{44})u_{xx} + c_{44}w_{xx} + c_{13}w_{xz} + e_{13}\phi_{xx} + e_{33}\phi_{zz} = \rho \frac{\partial^2 w}{\partial t^2},$$

$$(e_{13} + e_{15})u_{xx} + e_{15}w_{xx} + e_{33}w_{xz} - \kappa_{11}\phi_{xx} - \kappa_{33}\phi_{zz} = 0. \quad (3)$$

By introducing the displacement potential functions $\varphi$ and $\psi$, the equilibrium equations in the elastic substrate are simplified as

$$\nabla^2 \varphi = \frac{1}{c_p^2} \frac{\partial^2 \varphi}{\partial t^2}, \quad \nabla^2 \psi = \frac{1}{c_s^2} \frac{\partial^2 \psi}{\partial t^2}, \quad (4)$$

with $c_p = \sqrt{(\lambda + 2G)/\rho^e}$ and $c_s = \sqrt{G/\rho^e}$, where $\rho$ and $\rho^e$ are the mass densities of the piezoelectric medium and the elastic material, respectively.

2. Solution of the problem. As is well known, the above problem of wavy scattering can be treated as the superposition of the scattering problem of the incident wave in the same structure having no crack and the problem where the incident wave is applied to the crack surface but there is no incident wave in the far field. For the first sub-problem, the stresses induced by the incident wave at the crack position can be obtained easily and expressed in the following form:

$$\sigma_{xx}^e(x,t) = \tau_0(\omega,k)e^{i(kx-xt)}, \quad \sigma_{zz}^e(x,t) = \sigma_0(\omega,k)e^{i(kx-xt)}, \quad (5)$$

where the superscript $c$ stands for quantities of the incident fields, $\tau_0(\omega,k)$ and