FINITE ELEMENT WEIGHT FUNCTION APPLICATION FOR A CRACKED DISK

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Abstract. A practical application of the weight function method is used in this paper in order to find values of the stress intensity factor for a cracked disk subjected to different loadings. The finite element method is used in order to obtain discrete values of the crack face displacement in a reference loading case, namely inertia forces due to uniform rotation. These values are interpolated and a general expression of the displacements is obtained, which is further used to determine the stress intensity factor in this case. With the weight function equation, stress intensity factors for other loadings are obtained and the results are compared with those reported by other authors. Very good agreement was obtained, showing thus the reliability of this approach.

1. Introduction. The weight function method, introduced by Bueckner (1970) was extensively used in fracture mechanics for the computation of the stress intensity factor in cracked structures under elastic loadings. To apply this method, it is necessary to know a complete solution (the stress intensity factor and the displacements of the crack faces) to a crack problem for a loading system, called “reference loading”. Using these results, one may obtain the solution for the stress intensity factor for the same crack configuration with any other loading. With the co-ordinate system from Figure 1, Rice (1972) showed that, if the reference stress intensity factor $K_p(a)$, and the reference crack face displacements $u_p'(x,a)$ are available, the Mode I weight function can be determined from

$$h_I(x,a) = \frac{E'}{K_p(a)} \frac{\partial u_p'(x,a)}{\partial a}$$

(1)

where $E' = E$ (for plane stress) or $E' = E/(1 - \nu^2)$ (for plane strain), $E$ being the elastic modulus and $\nu$ – Poisson’s ratio.

An important feature of the weight function is that it depends only on the geometry of the body and the crack, being independent on the loading. Once the weight function for a given cracked body is known, the stress intensity factor for that
crack tip can be computed directly for any surface and body force loading, using the equation

\[ K = \int_0^a \sigma(x) \cdot h(x, a) \, dx \]

In equation (2), \( \sigma(x) \) is the crack line stress distribution in the uncracked body due to the loading for which the stress intensity factor is calculated. A weight function approach using the finite element method was proposed by Sha and Yang (1986). In their work, the reference stress intensity factors, the partial displacement derivatives and then the explicit nodal weight functions for the reference case were obtained with the virtual crack extension technique, coupled with singular elements. Once the explicit nodal weight functions were determined, a cubic spline interpolation technique was used for obtaining a general expression of the weight function for the structure of interest. The procedure is very accurate, but it is not easy to apply due to the fact that additional subroutines are to be added to the finite element code for the evaluation of the crack face profile derivatives and explicit nodal weight functions.

Another solution involving the finite element method was proposed by Pastram and de Castro (1998). In this technical note, an application of this approach is shown for the case of a hollow disk having an internal radial crack and subjected to different loadings. The obtained results are compared with those reported by Sha and Yang (1986) and it is shown that similar accurate results can be obtained with a simpler technique.

**2. Description of the method.** In the method reported by Pastram and de Castro (1998), the expression of the crack face displacements is taken in the form proposed by Petroski and Achenbach (1978). Their equation can be written in a general form as: