Dynamic Sampling Applied to Problems in Optimal Control

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Abstract. Dynamic sampling utilizes the option of varying the sampling rates according to the situation of the systems, thus obtaining procedures with improved efficiencies. In this paper, the technique is applied to a typical problem in optimal control theory, that of tracking and controlling the position of an object. It is shown that the dynamic sampling results in a significantly improved procedure for this case, even when applying a suboptimal policy which can be analyzed in closed form.

Key Words. Brownian motion, diffusion processes, observers, dynamic sampling, optimal control.

1. Introduction and Summary

Consider a typical problem in control theory, such as a tracking or regulator problem. As a specific illustration, we study the problem of tracking and controlling an object using a radar system. The trajectory is governed by a system of differential equations depending on the specification and the control applied. Our goal is to keep the object on path, taking the cost of the control into account. Our knowledge regarding the current state of the object is obtained by continuous tracking via the radar system.

The main difficulty of the tracking system is the presence of noise, and we make the distinction between two types of noise. The first is a noise affecting the actual movement of the object and the tracking system, due to various factors such as unpredictable weather conditions or inadequate modeling. The second type is the measurement noise present in the tracking

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system, which results in an imprecise reading of the exact position. While both noises are present in the models studied in this paper, the main focus is on the second type. In the dynamic sampling formulation, it is assumed that this second type of noise can be adjusted at any time according to our will.

A common model, which is used in the theoretical as well as in the applied work, is that the noise is a Gaussian white noise with fixed variance coefficient which depends on the characteristics of the tracking system; see, e.g., Refs. 1–3. In a multitracking system, the variance is inversely proportional to the amount of resources devoted to the particular object. This merely reflects the fairly trivial fact that, the more resources allocated, the better the performance; see Ref. 4.

The basic idea studied in this paper is the following. Rather than allocating a fixed amount of resources to each tracked object, apply a dynamic allocation procedure. Using this approach, the allocation is under our control and can be changed any time according to actual need. Thus, when the location of a given object is known fairly well, most of the resources may be shifted to the tracking of the other objects.

Mathematically, we study a somewhat simplified version of this problem in which a single object is being tracked; but rather than having a fixed measurement rate, this rate is under our control and a cost (or constraint) is associated in accordance with the rate selected. Thus, in this model, the tracking intensity may be selected at any given instant according to all current known information at that time. In particular, the intensity may depend on the current estimated state of the object and the variance of the estimate. The surprising result is that, under the optimal strategy, the sampling rate may be infinity. This may be not too practical, but we believe that this is a fair mathematical approximation to what can be done in practice when most of the resources are devoted to the particular unit under study.

A review of the results for the basic model (with a fixed sampling rate) is given in Section 2. The dynamic sampling formulation of the problem, as well as some basic conclusions, is presented and analyzed in Section 3. In Section 4, we propose a suboptimal sampling-and-control policy which can be analyzed in closed form. The performance of this policy is then compared to the fixed-rate policy in Section 5, and it is shown that the improvement obtained by dynamic sampling is indeed most significant. For ease of reference, we include a short Appendix with a summary of the basic results for the stochastic processes needed in the analysis.

It should be emphasized that the main feature of the dynamic sampling method is that it provides a highly improved performance without using any additional physical resources.